

# SELLING AND PRICING ON ONLINE OPAQUE CHANNELS

A Dissertation

Presented to the Faculty of the Graduate School  
of Cornell University

in Partial Fulfillment of the Requirements for the Degree of  
Doctor of Philosophy

by

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January 2012

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# SELLING AND PRICING ON ONLINE OPAQUE CHANNELS

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Cornell University 2012

Hotwire and Priceline, unlike other online travel sales channels such as Expedia, Travelocity and Orbitz, offer customers opaque products with aspects of the service provider concealed until the transaction has been completed. Selling on these opaque channels has become popular in service selling as it allows firms to sell their differentiated products at higher prices to regular brand loyal customers while simultaneously selling to non loyal customers at discounted prices.

This dissertation investigates how to optimally price on opaque channels while selling a fixed inventory over a finite horizon. This study also examines impacts on a firm's demand and profits by using opaque selling in addition to regular selling from both analytical and empirical perspectives. An online choice experiment is designed to understand customer preferences and trade offs while choosing among different online distribution channels.

## **BIOGRAPHICAL SKETCH**

Xie, Xiaoqing was born to two mathematics professors in Loudi, Hunan, China. She attended middle school and high school at the Loudi No. 1 middle school, Loudi, Hunan, China.

A natural interest in the mathematical and analytical world led her to study Mathematics as an undergraduate at Hunan University in Changsha, Hunan, China. She then went to pursue a master degree in Mathematics at University of Western Ontario in London, Ontario, Canada.

In 2006, she entered the School of Hotel Administration at Cornell University and conducted her doctoral research under the guidance of Drs. Chris Anderson, Rohit Verma, and Amr Farahat. She was majoring in Operations Management with special interests in dynamic pricing, revenue management and consumer choice modeling.

This thesis is dedicated to my parents.

## ACKNOWLEDGEMENTS

First of all, I am deeply grateful to my Ph.D. supervisor Dr. Chris Anderson for his guidance and patience through my dissertation writing. I very appreciate his support, great advice and encouragement on my research, which made my doctoral studies at Cornell smooth, enjoyable and productive. His dedication and enthusiasm to work and life inspired me to be a great professor and academic researcher.

I would also like to express my sincere gratitude to Dr. Rohit Verma for his help, advise and counselling on my learning of choice modeling and empirical study, which opened a whole new door for my research. I am very grateful for his kindness, expertise and support on my dissertation work and all these years of doctoral studies at Cornell.

I would also like to thank Dr. Amr Farahat for his kind patience and insightful advice on my research and dissertation writing. I would also like to thank professors who ever gave me lectures and valuable advice on research and life at both Cornell and UWO, and especially to Professors Dan Christensen, Henning Rasmussen, and Matt Davison. Special thanks are also going to my friends and colleagues whom I have had the pleasure of meeting at both Cornell and UWO.

Most importantly, I would like to take this opportunity to give my deepest thank to my parents for their endless support and encouragement over the years. This thesis is dedicated to them.

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## CHAPTER 1

### INTRODUCTION

This chapter introduces the main concept *Opaque Selling* discussed in the literature, outlining the goals this study would like to achieve and finally describes the organization of the remaining chapters.

#### 1.1. Opaque Selling and Objectives of the Study

The pricing of services (rooms, rental cars, airline seats, etc...) online has dramatically changed how service firms reach customers, with online travel sales now exceeding offline (or traditional sales channels). Adding online selling channels provides firms an opportunity to expand the market and achieve a finer consumer segmentation (Zettelmeyer 2000, Geysken et al. 2002). Hotwire and Priceline, unlike other online travel sites such as Expedia, Travelocity and Orbitz, offer customers opaque products (e.g. hotel rooms, flights and rental cars) with aspects of the service provider concealed until the transaction has been completed. We refer those traditional online travel sites like Expedia, Travelocity and Orbitz as the *regular full information channels*. The channels like Hotwire and Priceline are termed as *opaque channels*.

Specifically, for instance, a customer purchasing a hotel room through Hotwire can only specify check-in/out dates, a subarea within a city and a star rating. Customers do not know the identity or exact location of their nonrefundable choice property until after purchase, but the price is still posted to be seen. In this study, we will further refer the opaque channels like Hotwire

as the opaque posted channels. Similar to the opaque posted price model of Hotwire, Priceline offers opaque services but without posted prices. On Priceline, consumers post bids for the opaque service and then have to wait for the service provider to accept to reject their offer. These type of opaque channels are referred as the opaque bidding channel in the study.

The opaque channels naturally segment customers as regular full price paying customers desiring to stay at the hotel of their choice with full cancellation flexibility are unique from those willing to purchase the non-refundable but discounted opaque product at the unknown service provider. Revenue management is a revenue-maximizing practice that firms implement to segment the market by pricing into customers with heterogeneous valuations on the product. Thus, opaque selling can be viewed as a new revenue management technique (Jiang 2007). Opaque selling enables firms to facilitate the price discrimination and expand the market, which lead to making more profits than just using regular selling. These advantages has also popularized online opaque channels and stimulated the growing interest on opaque selling in the academic literature. The goal of this dissertation is attempting to fully understand the following:

- Connection between opaque selling and Revenue Management,
- Opaque selling mechanism and relative academic literature,
- Optimally pricing on a typical opaque channel while selling a fixed inventory over a finite horizon,
- Pricing and market segmentation while selling on both opaque and regular channels facing strategically behaving customers,
- Profit impacts from using opaque channels in addition to regular channels,

- Customer preferences and trade offs while choosing among different on-line distribution channels

Overall, I feel this dissertation provides a very strong contribution to service pricing by giving a deep insight into the role of opaque pricing for services. Although the study is conducted in a hotel context, it definitely opens the door for the use and adoption of opaque pricing across other industries.

## **1.2. Overview of Chapters**

The introductory chapter one is followed by Chapter 2, which provides a detailed background discussion for this study. Through reviewing the three main related streams of research: Revenue Management (RM), customer behavior in RM, online opaque selling; this chapter well positions the study in the literature and highlights its uniqueness and academic contributions.

On opaque channels, a differentiated good is transformed into a commodity. Optimal pricing becomes more important while selling through opaque channels. Chapter 3 uses a dynamic programming model to illustrate how a service firm can optimally set opaque prices while selling a fixed stock of inventory over a finite horizon with Poisson arrivals. The demand for the firm is modeled as a function of time, its own price and those of its competitors captured by using a nested logit model in combination with a logistic regression model. This study is motivated by a unique hotel booking and shopping data from Hotwire.com.

Chapter 4 models a firm selling a product via three channels: a regular full information channel, an opaque posted price channel and an opaque bidding

channel while facing strategically behaving customers. This chapter illustrates how opaque channels naturally segment consumers as well as how firms should use and price into these channels as a function of the degree of their opacity. This chapter also discusses the segmentation and policy changes induced by capacity constraints. It is shown that simultaneously selling through regular and opaque channels even in the presence of tight capacity constraints helps firms to segment consumers, differentially pricing into different willingness to pay segments and improve revenues (over the absence of opaque pricing).

Chapter 5 examines customer preferences among multiple online distribution channels by outlining the development and analysis of an online choice experiment. The experiment is developed from the standpoint of a hotel using Expedia.com or Marriott.com as a regular full information channel in concert with an opaque posted price channel and/or an opaque channel with bidding. A Multinomial Logit (MNL) model is employed to analyze the experimental data and measure the customers trade offs between price and other attributes of the product, as well as quantify the profit impacts from using opaque channels in addition to regular full information channels.

Chapter 6 summarizes the dissertation, discusses the limitation of the work and provides potential future research directions.

## CHAPTER 2

### BACKGROUND AND RELATED RESEARCH

#### **2.1. Revenue Management**

Revenue management (or yield management), originating from solving over-booking and cancellations problems (Rothstein 1971, 1974), has become a common practice to improve revenue in the airline industry since its U.S.A. deregulation in 1978. Revenue management (RM) has been implemented most widely in airlines, hotels and rental car industries and they share similar characteristics such as high start-up costs, perishable capacity, short selling horizons, demand being stochastic and price sensitive. Boyd and Bilegan (2003) elaborated the equivalence and similarity between the mathematical models formulated from the RM problems in the hotel and airline industries. For instance, different levels of hotel room rates based on rooms' physical characteristics correspond to different classes of airline fares for first, business, and coach cabins. A network airline RM problem considers a route comprising several individual flight legs, which is analogous to a hotel problem with multiple night stays. The hotel and airline RM problems are quite similar, but are not completely identical. RM at an airline is applied in a centralized manner, but most hotels are decentralized with regard to RM, with individual properties usually controlling RM systems (Anderson and Xie 2010). Kimes (1989) outlines the industry characteristics amenable to the application of RM, and then gives a general overview of RM practice in the hotel industry as well as discusses implementation issues for hotel managers. Hotel industry is the background for the research in this dissertation, however, the main insights and results could be generated to other



industries without much effort given their similarities.

Because of the success of the practice of RM in those traditional industries, over the years, RM has spread out into other non traditional industries such as restaurants, cruise lines, casinos, cargo, golf courses, health care, internet services, retailers etc. (Kimes 2003, Bitran and Caldentey 2003). Revenue management is essentially a tool implemented to maximize revenue by selling the right products to the right customers at the right time (Kimes 1989). Talluri and van Ryzin (2004b) provide a comprehensive overview of every aspect of revenue management. Chiang, Chen and Xu (2007) focus on reviewing the research on revenue management after 1999 and classify the published work using taxonomy. Chiang, Chen and Xu (2007) not only discuss the RM practice in a wide range of different industries and provide a thorough discussion of primary RM problems, but also touch on some important related issues about RM and give some future research directions in the area. Anderson and Xie (2010) provide a comprehensive review on the RM-related research articles published in the *Cornell Quarterly* (an applied academic journal widely considered to be the top-tier journal in the discipline of hospitality, travel and tourism) in the past twenty five years. Although the review focuses on the research published on one journal, it documents the history of RM expanding from tractional industries into non traditional service industries, with an eye to future developments in RM. Table 2.1, based on Chiang, Chen and Xu (2007) and Anderson and Xie (2010), summarizes the papers dealing specifically with hotel-related RM.

Revenue management problems mainly include these four areas: pricing, inventory control, overbooking and forecasting. Here we will be focusing on reviewing the research articles on pricing and inventory controls and highlighting

Table 2.1: Hotel Revenue Management

Year	Reference	Year	Reference
1988	Orkin	2001	Kimes and McGuire
1988	Lefeverand	2001	Kimes and Wagner
1989	Relihan	2002	Baker et al.
1989	Kimes	2002	Barth
1989	Lambert, Lambert, and Cullen	2002	Choi and Kimes
1990	Dunn and Brooks	2002	Goldman et al.
1992	Hanks, Cross, and Noland	2002	Toh and Dekay
1993	Lieberman	2003	Baker and Collier
1994	Kimes	2003	Orkin
1995	Bitran and Mondschein	2003	Rannou and Melli
1995	Weatherford	2003	Varini et al.
1996	Bitran and Gilbert	2003	Weatherford and Kimes
1997	Cross	2004	Anjos et al.
1997	Hadjinicola and Panayi	2004	Chen and Freimer
1997	Zheng and Caneen	2004	Kimes
1998	Kimes et al.	2004	Liu
1998	Orkin	2004	Mainzer
1999	Baker and Collier	2004	Okumus
1999	Quain, Sansbury and Quinn	2004	Schwartz and Cohen
2000	Choi and Cho	2004	Vinod
2000	Huyton and Thomas	2005	Choi and Mattila
2000	Jones	2005	Jain and Bowman
2000	Kimes	2005	Lai and Ng
2000	Main	2005	Koide and Ishii
2000	McMahon-Beattie and Donaghy	2006	Choi
2000	Noone and Andrews	2006	Choi and Mattila
2001	Elkins	2007	Rohlf and Kimes
2001	Weatherford and Kimes	2009	Cross, Higbie, Cross
2001	Weatherford, Kimes, and Scott		

the work that have bearing upon the dissertation.

### **2.1.1 Inventory Control**

After the deregulation in the early 70's and discounted fare classes were added, airlines were facing the problem of optimally allocate seats to different classes of demand in order to maximize the expected revenue or profit. This process of seeking the best mix of fare classes to maximize the airline revenue is called seat allocation, which is referred as the inventory control in a more general term in other industries. Ever since the early stage of RM, enormous research has been dedicated to this main component of revenue management, especially in the airline industry. Inventory control has progressed from relatively simple single-resource inventory control to more complicated network inventory control (Chiang, Chen and Xu 2007).

#### **Single-resource inventory control**

In the airline industry, single-resource inventory control is referred to as single-leg seat inventory control as indicated in Chiang, Chen and Xu (2007). Single-leg seat inventory control problems consider maximizing the revenues on a particular leg of a passenger's potential flight itinerary. The first useful result on the seat allocation problem was presented by Littlewood (1972). He proposed a well known simple seat allocation rule for flights with two nested fares, which is continuing to sell discount seats as long as discount fare equals or exceeds the expected marginal revenue of future sales in full fare class.

Belobaba (1989) generalized Littlewood's rule to a multiple fare revenue model called Expected Marginal Seat Revenue (EMSRa) model. EMSRa is Littlewood's rule applied sequentially in increasing fare order and only optimal for two fare classes. Belobaba (1987a) gives a good overview of the work done till 1987 in seat inventory control in the airline industry. EMSRa is later refined by Belobaba into EMSRb (Belobaba and Weatherford 1996). It is logically the same as Littlewood's rule but the expected marginal seat revenue from future sales is now a weighted average fare from higher classes.

Curry (1990), Wollmer (1992) and Brumelle and McGill (1993) independently showed the nonoptimality of Belobaba's EMSRa policy. For continuous demand distribution functions, Brumelle and McGill (1993) provided the optimality conditions in a form analogous to the EMSRa formula. Li and Oum (2002) noted that these three models and their optimality conditions for finding the optimal booking policies are actually equivalent.

Table 2.2 chronologically lists single-leg seat inventory control research McGill and van Ryzin (1999). Chiang, Chen and Xu (2007) reviewed some representative articles that modeled single-resource inventory control problems in different industries other than airlines after 1999.

All EMSRa, EMSRb, and Littlewood's rule assume that it is a single flight leg with no batch bookings, lower fare classes book before higher fare classes, fare classes are mutually independent, no cancellations or no-shows. These rules are usually used in a static fashion. Lautenbacher and Stidhum (1999) divided the previous papers on seat inventory control in airline industry into two categories: dynamic model papers and static model papers. Dynamic and static are the terms to present different customer arrival patterns. For instance, the papers by

Table 2.2: Leg Based Seat Allocation

Year	Reference	Year	Reference
1972	Littlewood	1992	Stone and Diamond
1973	Bhatia and Parekh	1992	Sun
1976	Mayer	1992	Wollmer
1977	Ladany and Bedi	1993	Weatherford et al.
1978	Hersh and Ladany	1993	Lee and Hersh
1982	Wang	1993	Brumelle and McGill
1982	Buhr	1994	Weatherford
1982	Richter	1994	Shaykevich
1983	Titze and Riesshaber	1994	Young and Slyke
1985	Simpson	1995	Bodily and Weatherford
1986	Alstrup et al.	1995	Robinson
1986	Kraft et al.	1996	Belobaba and Weatherford
1986	Pratte	1997	Brumelle and Walczak
1986	Wollmer	1998	Kleywegt and Papastavrou
1985	Gerchak et al.	1998	Li and Oum
1987	Gerchak and Parlar	1998	Li
1989	McGill	1998	van Ryzin and McGill
1989	Belobaba	1998	Zhao and Zheng
1989	Pfeifer	1999	Subramanian et al.
1990	Brumelle et al.	1999	Lautenbacher and Stidham
1991	Weatherford		

Littlewood (1972), Belobaba (1989), Brumelle and McGill (1993), Wollmer (1992), Curry (1990) and Robinson (1995) are static model papers; the papers by Hersh and Ladany (1978), Lanany and Bedi (1977), Lee and Hersh (1993) are dynamic model papers.

Dynamic programming (DP) is used to solve the dynamic models that relax some of the assumptions required for the Littlewood's rule, EMSR and optimal policies discussed above and allow some real world factors such as cancellations, overbooking, batch bookings and interspersed arrivals. Hersh and Ladany (1978), Lanany and Bedi (1977) were the early attempts to use dynamic programming to solve the problem of allocating seats on a two segment flight. Gerchak et al. (1985), one of the first examples of applying revenue management in non traditional industries, formulated a DP to solve a RM problem in a bagel shop. The problem was equivalent to an airline facing two fare classes customers arriving in a stochastic process. Lee and Hersh (1993) extended Gerchak et al. (1985) to multiple fare situations and broke the decision horizon into numerous stages allowing only one request per period. Subramanian et al. (1999) extended Gerchak et al. (1985) in a way that they not only relaxed the assumption of no batch bookings, but also allowed overbooking, cancellations and no shows.

As it was indicated by McGill and van Ryzin (1999), DP formulations, especially stochastic ones, are well known for their unmanageable growth in size when real-world implementations are attempted. Some approximation methods for solving DP and stochastic programming problems in RM were developed in the 90's. The body of literature on approximate dynamic programming is relatively small and one can find good reference books by Bertsekas and Tsit-

siklis (1996), Birge and Louveaux (1997) and Powell (2007). Recently, dynamic programming approaches are also used in research attempts to incorporate network effects in seat allocation decisions.

### **Network Inventory Control**

Network inventory control addresses the inventory allocation problem when customers request a bundle of different resources. In the airline industry, this problem is also known as the origin-destination based seat allocation problem. In the hotel and rental car industry, network inventory control is used when customers request multi-night stays or multi-day rentals, respectively. The revenue considered in this kind of problems is the entire revenue from all connecting flights or all nights' stays or all days' rentals.

This dissertation is based on the assumption that customers only request one night stay and one room at a time (i.e. the single-resource), so we do not spend too much effort on reviewing this stream of work. McGill and van Ryzin (1999) provide a thorough development and review of network inventory control problems in the airline industry.

Chiang, Chen and Xu (2007) reviewed some research that addressed network inventory control problems mostly in the airline industry and one in the hotel industry, which are all published in the 2000's. These problems are mainly formulated into stochastic dynamic programming models and solved by the approximate methods that are discussed in the previous single-resource inventory control section.

Table 2.3, based on McGill and van Ryzin (1999) and Chiang, Chen and Xu

(2007), chronologically lists references for network inventory control research over the past three decades.

Table 2.3: Origin-Destination Based Seat Allocation

Year	Reference	Year	Reference
1982	D'Sylva	1993	Talluri
1982	Glover et al.	1994a	Talluri
1983	Wang	1995	Vinod
1985	Simpson	1996	Talluri and van Ryzin
1986	Wollmer	1997	Garcia-Diaz and Kuyumcu
1987	Belobaba	1999	Ciancimino et al.
1988	Dror et al.	1999a	Talluri and van Ryzin
1988	Smith and Penn	2001	Feng and Xiao
1988	Williamson	2001	Talluri
1988	Wysong	2002	de Boer et al.
1989	Simpson	2003	Bertsimas and Popescu
1989	Vinod	2004	El-Haber and El-Taha
1990	Curry	2004	Gallego, G. and Phillips, R.
1990	Vinod and Ratliff	2004	Möller et al.
1990	Vinod	2004	Pölt
1990	Wong	2005	Lai and Ng
1991	Phillips et al.	2007	Zhang and Adelman
1991	Vinod	2008	Liu and van Ryzin
1992	Williamson	2008	van Ryzin and Vulcano



### 2.1.2 Pricing

Pricing has become one of the hardest, yet most effective decisions that managers need to make to stimulate the demand and gain the most profits at a daily basis in an manufacturing or service company. Especially for industries which have perishable products facing uncertain demand, pricing becomes one of the main components of revenue management and has had substantial amount of attention from researchers over the decades. McGill and van Ryzin (1999) state that revenue management is the practice of controlling the availability and/or pricing of travel seats across different booking classes with the goal of maximizing expected revenues or profit in the airline industry. Bitran and Caldentey (2003) provide a good overview of the pricing research and compare results of different pricing models for revenue management.

The goal of pricing is how to optimally determine the price for various customer groups and vary prices over time in order to maximize revenues. McGill and van Ryzin (1999) also pointed out that the existence of differential pricing for airline seats is the starting point for revenue management, and price is generally the most important determinant of passenger demand behavior. In fact, as the two main tools of revenue management, pricing and seat inventory control are fundamentally equivalent or there exists a natural duality between them (Gallego and van Ryzin 1997). The distinction between them is whether we treat price or number of seats allocating to each fare class as the decision variable in the model. More specifically, if we can adjust the price of a single product continuously (dynamically), a booking class can be closed for sale by raising the price sufficiently high, i.e. 'shutting down a booking class can be viewed as changing the price structure faced by the customer'(McGill and van Ryzin

1999). By illustrating that the multi-product dynamic pricing problem studied in Gallego and van Ryzin (1997) and the inventory control problem of Lee and Hersh (1993) can be reduced to a common formulation and thus be treated in a unified manner, Maglaras and Meissner (2004) showed the equivalence between these two. Research on coordinated pricing and inventory decisions is surveyed by Chan et al. (2004) and Yano and Gilber (2003). Although these two areas are highly correlated, there have been very little published research on jointly inventory control/pricing decisions in the RM literature until recently (Botimer 1994; Li 1994; Weatherford 1994; Gallego and van Ryzin 1997; Federgruen and Hetching 1999; Feng and Xiao 2006).

Over the past decades there has been a rapid growth in internet sales channels and new technologies, which provides the companies/service providers the capability of adjusting their prices dynamically and easily according to the current inventory and demand level. Stochastic dynamic pricing models are used to maximize expected revenues while selling a fixed inventory over a finite horizon (Kincaid and Darling 1963; Gallego and van Ryzin 1994, 1997; Bitran and Mondschein 1997; Bitran et al. 1998; Fen and Xiao 2000a,b; Zhao and Zheng 2000). Demand is modeled as a Poisson process with a intensity that is price and/or time sensitive. Unlike most of the literature that assumes the intensity is a decreasing, or more particularly, a *regular* function of the price (Gallego and van Ryzin 1994), in Chapter 3, we let the intensity be dependent on time and capture the price effect through a discrete choice model and a logistic regression model. Thus, our demand model is the combination of a Poisson arrival process of customers and a particular selling process based on the firm's and its competitors' posted prices which explicitly characterizes the competitive forces in the market. As in Lee and Hersh (1993), we divide the selling horizon into

small decision periods of equal lengths such that there is at most one arrival request in each period. Despite of the difference in demand models, we have obtained the same nice properties about value functions and optimal pricing strategies. The value function is concave increasing function of the remaining selling time and inventory level. The optimal price is nondecreasing in the level of inventory and nondecreasing in time. It is generally very hard to find closed form solutions to these stochastic dynamic pricing problems, and even it is not easy to implement and control the numerical solutions in practice. Therefore, efforts have been made on finding approaches for solving a deterministic version of the problem to get an approximate optimal solution. Similar idea is used in this chapter where we solve a "less dynamic " version of the problem, which is varying prices daily instead of varying prices at each arrival. We compare the optimal prices and maximum expected revenues under these two frameworks and numerically show that dynamic pricing performs better in spite of its difficulty of implementation in reality.

Table 2.4 contains a chronological list of relevant articles in the revenue management context.

## **2.2. Customer Behavior in Revenue Management**

The majority of the work in revenue management literature assumes customer demand is exogenous given and completely independent of time, the availability of other products and some other factors. In the early stage, some approximate analyses of customer choice behavior were conducted to correct this unrealistic assumption for inventory control problems. They usually incorporate

Table 2.4: Pricing

Year	Reference	Year	Reference
1971	Kostrysky	2000a,b	Feng and Xiao
1981	Dolan and Jeuland	2000	Zhao and Zheng
1984	Reibstein and Gatignon	2001	Curry
1988	Lau and Lau	2001	Weatherford
1990	Oram	2003	Anderson and Wilson
1991	Weatherford	2003	Bitran and Caldentey
1992	Weatherford and Bodily	2003	Coteetal
1993	Oum et al.	2003	Elmaghraby and Keskinocak
1994	Borenstein and Rose	2003	Frank
1994	Botimer	2003	Bitran and Caldentey
1994	Carpenter and Hanssens	2004	Anjos et al.
1994	Gallego and van Ryzin	2004	Fleischmann et al.
1994	Inzerilli and Jaradiaz	2004	Xu and Hopp
1994	Li	2005	Aviv and Pazgal
1995	Kretsch	2005	Burger and Fuchs
1995	Oum	2005	Levin et al.
1995	Feng and Gallego	2005	McAfee et al.
1996	Dana	2005	Zhang and Cooper
1997	Bitran and Mondschein	2005	Zhou et al.
1997	Gallego and van Ryzin	2006	Maglaras and Costantinos
1998	Dana	2006	Maglaras and Meissner
1999	You	2007	Su
1999	Baker and Collier	2008	Levina et al.
1999	Desiraju and Shugan	2008	Liu and van Ryzin
1999	Federgruen and Heching	2008	Zhang and Cooper
2000	Feng and Gallego	2009	Levin et al.

the effects of "buy-up" (buying a higher fare when a low fare is closed) and "buy-down" (willing to pay a high fare instead buys an available low fare) into their demand models (Belobaba 1987a,b; Belobaba and Weatherford 1996). As one of the earliest attempts on considering customer behavior, Anderson and Wilson (2003) solved an optimal seat allocation problem where customers anticipate closed low fare classes having some probability of reopen again in the future. Their optimal protection limit set for each fare class is based on Belobaba's (1989) EMSR rule. They find that using standard pricing approaches can cause substantial losses in revenues while facing customer who strategically wait with the hope of lower prices later in time.

The growing use of the internet not only offers the firms an easy, inexpensive, and more effective way to change price dynamically, but also allows customers to behave more strategically when making purchase decisions. More specifically, the internet provides an opportunity for customers to gather information such as capacity availability and firms' pricing policies, as well as do comparison-shopping among different online travel sites or service providers. By using the information they have collected, customers may behave strategically by delaying their purchases with hopes of getting a better deal in the future, or pick an online travel site/service provider to make a purchase for their best interest. In other words, more and more customers attempt to strategically determine *when* to buy and *what* to buy while making their purchasing decisions.

Therefore, the literature on strategic customer behavior in RM can be categorized into two groups. The first group of research examines the behavior of customers who strategically time their purchase in response to firms' dynamic

pricing practices. These customers are conventionally termed as *strategic customers* in the literature. In contrast, the customers who make a one-time purchase decision upon their arrival are termed as *myopic customers* (Shen and Su 2007). The problem that firms are facing in most of the papers in this group of research is commonly referred as the inter-temporal pricing problem. Unlike the early exogenous demand models, now demand may depend on time and prices. The second group of papers studies customer choice behavior in multi-product revenue management settings. In particular, recently, there has been a growing interest in modeling consumer choice behavior in revenue management research by using discrete choice models. Demand in these models are now dependent on the characteristics of other available products such as price, star level, location, amenities and so on if it is in a hotel industry setting.

For a more thorough and complete review of customer behavior modeling in RM, please refer to Shen and Su (2007).

### **2.2.1 Inter-temporal Pricing facing *Strategic Customers***

Inter-temporal price discrimination has its origin in the economics literature on durable goods monopoly (Stokey 1979, 1981). Besanko and Winston (1990) study a game between a monopolistic seller and strategic customers and show the subgame-perfect equilibrium policy for the firm is decreasing prices over time (*price skimming*). There is no uncertainty and no capacity constraint in their model.

Recently, the research on dynamic pricing with strategic customers gained some significant attention in the revenue management literature. Most of the

work considers a monopolist who sells a finite perishable inventory over a finite time horizon (two periods). A number of papers consider mark down pricing mechanisms (Elmaghraby et al. 2008, Aviv and Pazgal 2008, Cachon and Swinney 2007, Gallego et al. 2007, Liu and van Ryzin 2007, Zhang and Cooper 2009). Elmaghraby et al. (2008) set prices decreasing according to a pre-announced schedule and assume strategic customers demand multi units. Aviv and Pazgal (2008) compare a pre-announced fixed-discount strategy with a contingent pricing policy and customers arrive in a Poisson process with deterministic declining valuations over time. They formulate the inter-temporal pricing problem into a game between the seller and customers and identify a subgame-perfect Nash equilibrium. Chapter 3 models a service firm pricing a fixed inventory over a discrete selling horizon. Although the demand is characterized by choice models, another dimension of strategic customer behavior that we will discuss in the next section, we have demonstrated that the optimal price is also decreasing over time.

On the contrary, some papers address that optimal prices should increase over time (Gallien 2006, Arnold and Lippman 2001, Das Varma and Vettas 2001). Several papers allow both markups and markdowns. Su (2007) models customers being heterogeneous in two dimensions: product valuation and degree of patience (waiting costs). The demand is modeled as a continuous and deterministic flow with a constant rate. Levin et al. (2005) modeled a stochastic dynamic game between a firm and customers who attempt to maximize the expected utility by timing their purchases strategically. By using numerical example they show that ignoring strategic customer behavior may lead to lower total revenues. Ovchinnikov and Milner (2010) consider a multi-period setting where the model allows for both stochastic consumer demand and stochastic

waiting.

Some other relevant work can also be found in advance selling literature in Marketing field (Xie and Shugan 2001; Tang et al. 2004).

Table 2.5 summarizes the relevant research in this section.

Table 2.5: Inter-temporal Pricing facing *Strategic Customers*

Year	Reference	Year	Reference
1979	Stokey	2006	Asvanunt and Kachani
1981	Stokey	2006	Levin et al.
1990	Besanko and Winston	2007	Ahn et al.
2001	Arnold and Lippman	2007	Cachon and Swinney
2001	Das Varma and Vettas	2007	Gallego et al.
2001	Xie and Shugan	2007	Liu and van Ryzin
2003	Anderson and Wilson	2007	Su
2004	Tang et al.	2008	Aviv and Pazgal
2004	Xu and Hopp	2008	Elmaghraby et al.
2005	Levin et al.	2009	Levin et al.
2005	Zhou et al.	2009	Zhang and Cooper
2006	Gallien	2010	Ovchinnikov and Milner

### 2.2.2 Customer Behavior with Multi-product Choices

As mentioned previously that the traditional models of revenue management assume demand for each product is simply not influenced by the availability of other products. Recently, there has been a growing interest in using discrete



choice models of customer purchase behavior as an alternative to the independent demand models in revenue management research. Discrete choice models have been applied most commonly in marketing literature where they are used to study consumer choice behavior and understand consumer's brand preferences, market structure as well as product attributes. Coretjens and Gautschi (1983) provide a general survey of discrete choice models in marketing and a systematic introduction to discrete choice modeling theory. During the past 40 years, the multinomial logit (MNL) model has been the most popular discrete choice model used by marketing researchers. MNL models require alternatives to have the same relative probability of being selective independent of the choice set, i.e. the so called Independence of Irrelevant Alternatives (IIA) property. Nested logit model is one of the approaches employed to relax the IIA assumption (McFadden 1981; Ben-Akiva and Lerman 1985), which has also been commonly implemented by marketing researchers.

Over the past decade, choice based models have been introduced in the revenue management literature where both price and inventory decisions need to be made. Andersson (1998) using data from Scandinavian Airlines System uses a MNL model to estimate passenger preferences among buying up, being recaptured and deviating to another airline. Talluri and van Ryzin (2004b) study consumer choice behavior among multiple fare classes on a single-leg flight using a MNL model. They formulate the optimal seat allocation policy as a dynamic program (DP). This formulation is extended by Liu and van Ryzin (2008) to a network case, in which their DP problem is approximated by a deterministic linear programming problem. Similarly, Gallego et al. (2004) also use a deterministic approximation to solve a network RM problem where newly defined *flexible products* are chosen endogenously by customers. Zhang and Adelman

(2007) also consider a network revenue management problem where consumer choice behavior is explicitly modeled by a MNL model. They claim that they provide a better approximation to the resulting DP model of Liu and van Ryzin.

The study in Chapter 3 is similar to Talluri and van Ryzin (2004b), Liu and van Ryzin (2008), Zhang and Adelman (2007) in that we all use a discrete choice model to characterize consumer choice behavior and formulate the revenue management problem as a dynamic programming problem. However, we solve the resulting stochastic DP problem versus simply obtaining a deterministic approximation or assuming a priori optimal policy. This difference is due to the fact that we use price as the decision variable in the DP instead of deciding which subsets of fare classes should offer to customers. Given the equivalence between pricing and inventory allocation problems discussed before, it is easy to see the relevance between the previous papers and this work. To our knowledge, this work is the first attempt in the literature to characterize customer choice behavior by using a NL model versus a MNL model.

Zhang and Cooper (2005) study a seat allocation problem when customers dynamically choose among multiple parallel flights. In stead of considering seat allocating decisions, Zhang and Cooper (2007) consider pricing decisions when customers face the same choice situation as in Zhang and Cooper (2005). Similar to our model setting, they also assume that there is at most one arrival in each period and the arrivals follow a non-homogeneous Poisson process. Although they do not model customer choice behavior by employing discrete choice framework, the same as ours that they consider customers choose products from different inventory resources. Whereas, other papers reviewed above examine customer choice among different products drawing from a common

pool of resource (Shen and Su 2007).

Similar to Andersson (1998) and Algers and M. Besser (2001), Vulcano et al. (2008) perform an empirical choice model study of estimation and optimization on actual airline data from a major U.S. airline. Ratliff and Rao (2007) is another one of the few empirical studies on how choice behavior impacts revenue management decisions. Both the simulated and real data in all the work above are using sales transactions from one service provider. In contrast, our data contains complete market level booking and shopping records, which allows us to use the traditional maximum likelihood method (MLE) estimation. As far as we know that this work is also the first choice-based revenue management research applied to the hotel setting, whereas almost all the prior literature were focused on airline revenue management systems. There are also several papers that study customer choice behavior in health care revenue management problems (Gupta and Wand 2005, Green et al. 2006).

Summary of the related research in this section is given in Table 2.6.

### **2.3. Online Opaque Selling**

Opaque selling is implemented to naturally segment customers as regular full price paying customers desiring to stay at the hotel of their choice with full cancellation flexibility and discounted price paying customers are willing to purchase the non-refundable opaque product at the unknown service provider. One can view opaque selling as a new revenue management technique since revenue management is essentially a revenue-maximizing practice that firms use to segment the market by pricing into customers with heterogenous valuations on the

Table 2.6: Customer Behavior with Multi-product Choices

Year	Reference
1998	Andersson
2001	Algers and Besser
2004	Gallego et al.
2004b	Talluri and van Ryzin
2005	Gupta and Wand
2005	Zhang and Cooper
2006	Green et al.
2007	Ratliff and Rao
2007	Zhang and Adelman
2007	Zhang and Cooper
2008	Liu and van Ryzin
2008	Vulcano et al.

product.

Opaque selling has recently started to receive interest in the academic literature, most of the early research has focused on models similar to Priceline's NYOP bidding mechanism where customers post bids for opaque services. Several papers consider a monopolist firm uses a Priceline or Priceline-like NYOP mechanism. Their research issues include setting optimal minimum acceptable price and/or bidding rules, finding implications of customer expectations and behavior etc. (Anderson 2009; Fay 2004; Hann and Terwiesch 2003; Terwiesch, Savin and Hann 2005; Ding et al. 2005; Fay 2008a; Wilson and Zhang 2008; Fay and Laran 2009).

### **2.3.1 Selling on Posted Opaque Channels**

Research on posted price opaque mechanisms has been focused on providing a rationale for opaque selling and examining the impact of opaque selling on the market while comparing selling through the regular full information channel (Fay 2008b; Jiang 2007; Shapiro and Shi 2008; Jerath et al. 2007). As far as I know that there is no published research in opaque selling literature that is very close to the study in Chapter 3. In this chapter, a nested logit model is used in combination with logistic regression and dynamic programming to illustrate how a service firm can optimally set prices on Hotwire-like posted opaque channel. The work is motivated by a unique set of booking and shopping hotel data in Washington DC and is obtained from Hotwire.com. It is also the first research effort that attempts to capture how a service firm is able to make a sale on Hotwire under a competitive market and characterize the price competition by using a choice model and logistic regression model.

### **2.3.2 Simultaneously Selling on both Opaque and Regular Channels**

Most relevant stream of literature to Chapter 4 is on using opaque channels in a multi-channel selling environment. Fay (2008b) uses a traditional Hotelling model to study a game between two service providers selling products to two types of customers (*brand-loyals* and *searchers*) on both an opaque posted price channel and a traditional distribution channel. Fay shows that opaque selling benefits the monopoly service provider when customers have heterogeneous values for products, which is consistent with our main insight. In stead of exoge-

nously giving two customers segments, *brand-loyals* and *searchers* in Fay 2008b, our model allows all customers to segment themselves by maximizing their surpluses on each channel.

Similar to Fay (2008b), Jiang (2007) also develops a Hotelling type model to illustrate how a firm should price on regular full information channels versus opaque channels. Similar to us, Jiang considers a monopoly firm and so neglects competition in both the regular and opaque markets. This simplification enables the study to be focused on investigating when to implement opaque selling. Shapiro and Shi (2008) extend the model of Fay (2008b) to  $N$  firms with the number of firms indicating the degree of opacity - uncertainty in knowledge of service provider increases with number of firms. We are the first study that explicitly models this degree of opacity and uses it to demonstrate how customers value opaque products compared to full-information products. Jerath et al. (2010) compare opaque selling with last-minute direct selling and obtain the conditions under which opaque selling is preferred. A two period duopoly game is examined, and so consumers are assumed to strategically time their purchases along with choosing the best channel to buy. Unlike most of the related research including ours, Jerath et al. (2010) considers the situation where demand is uncertain and constrained by capacity.

So far, the papers we have reviewed have focused on comparing selling on posted opaque channels like Hotwire and on regular full information channels like Expedia. There are a few papers in the literature that consider selling on different channels including the Priceline like NYOP channel. Wang et al.(2009) develop a two period game theoretic model of a supplier using both regular posted price full information channels as well as a NYOP channel to reach het-

erogeneous customers. They first partition customers into two groups, business and leisure travellers, and their leisure travellers will then be divided into segments according to their willingness to pay for the service, which is assumed to be uniformly distributed over  $[0, 1]$  as in our model. However, different from us, it is assumed that the customers can not return to the regular posted-price channel to purchase the service if their bid on NYOP channel fails.

Fay (2009) examines a game between two firms, and each one can sell either through the NYOP mechanism or the posted price channel like Hotwire. In his model, customers have different frictional costs on bidding on a NYOP retailer but have the same reservation value for products, whereas we segment customers based on their heterogeneous products valuations. Cai et al. (2009) investigates the potential benefits to a NYOP retailer by adding a retailer-own list-price channel. They compare two bidding scenarios, single-bid and double-bid, in both single-channel and dual-channel situations. Their model setup is close to ours in a way that customers are allowed to come back to the list-price channel if their bid is rejected (also in Ding et al. 2005). However, instead of endogenously partitioning themselves into segments by choosing their preferred channels to buy as modeled in our study, their customers are segmented into two groups where one group buys from list-price channel only and the other one is willing to bid.

Priceline's NYOP mechanism is commonly viewed as a reverse auction (Ding et al. 2005; Fay and Laran 2009; Fay 2009). While there is an extensive body of research on the use of auctions, very little of this research looks at the simultaneous use of auctions and posted prices. Firms can use auctions to reach customers whom may not otherwise purchase, as posted prices may be

too high. Conversely auctions potentially dilute revenues as customers willing to pay posted (full prices) may purchase (at lower prices) via the auction. The opaque nature of Priceline's NYOP model helps to avoid this dilution. Etizon, Pinker and Seidmann (2006) is one of the few auction related papers that looks at the simultaneous use of auctions and posted prices. Similar to our development they look at a firm with excess supply facing consumers who strategically choose to purchase at posted prices or bid (resorting to posted prices if their bid fails). Caldentey and Vulcano (2004) study a RM problem where the seller manages simultaneously both the multiunit auction and the list price channels with customers arriving in a Poisson process. They also assume customers endogenously segment themselves by maximizing their own surpluses. Different from our model, consumers do not face any product opacity with the auction but do incur a waiting cost associated with bidding. Van Ryzin and Vulcano (2004) look at firm using posted prices as well as an auction mechanism, unlike our model of endogenous channel choice (strategic customers similar to Etizon et al.) they assume separate streams of customers to each channel with the seller deciding on inventory allocation across the channels.

There are some other closely related research which consider various forms of product differentiation and price discrimination. Fay and Xie (2008) define a new form of product termed as *probabilistic goods* under which customers have a probability of getting any one of several distinct goods. Customers do not know the identity of the good until after the purchase, which is analogous to opaque selling. By introducing *probabilistic goods*, firms are able to reduce the mismatch between capacity and demand under the circumstance of uncertain customer preferences structure. Another similar concept *flexible product* was introduced by Gallego and Phillips (2004) and Gallego et al. (2004). By selling a *flexible*



*product*, firms have the flexibility of assigning one out of a set of alternative products to the customer if there are excess inventories for that product. Like probabilistic goods, flexibility products are also offered at a discount, which provides a new dimension for firms to segment a market as well as improve capacity utilization. An opaque product is also somewhat comparable to *Damaged goods* in Deneckere and McAfee (1996) because products available on opaque channels are lack of some critical information before the transaction is finished and also may not be refunded for free. Deneckere and McAfee (1996) show that firms could have a Pareto improvement in profits through selling intentionally damaged goods along with the rest high-quality goods to heterogenous consumers. They model a monopolist firm segments the market in two cases by offering damaged goods. The first case is there are two distinct types of customers who buy either high or low quality goods only, whereas in the second case consumers are having continuous value on goods. Our model setting fits in their second case and so market is segmented endogenously. Versioning literature (Varian 2000) and multi-channel competition literature (Chiang et al. 2003; Coughlan and Soberman 2005; Zettelmeyer 2000) is also relevant.

In Chapter 4, we develop a stylized model of consumers looking to acquire travel services through either full information or opaque channels (both posted price and bidding). Consumers choose their channel or sequence of channels (in the case of bidding first followed by posted prices) that maximizes their surplus. This study is unique from the previous reviewed literature in that it is the only paper that investigates a firm using two opaque (posted and bidding) channels simultaneously with regular full information posted price channels. Second, most of the prior research assumes two or more exogenous customer segments (i.e. business and leisure) with the opaque channels targeted at the leisure or

price sensitive segment; whereas we develop endogenous consumer segments where consumers choose the channel of their choice by maximizing their surplus. We also discuss the segmentation and policy changes induced by capacity constraints. We show that simultaneously selling through regular and opaque channels even in the presence of tight capacity constraints helps firms to segment consumers, differentially pricing into different willingness to pay segments and improve revenues (over the absence of opaque pricing).

To our knowledge, the research work in Chapter 5 is the first empirical study to understand opaque selling along with regular selling through experimental choice analysis. From both analytical and empirical perspectives, studies in Chapter 4 and 5 show that appropriately using opaque selling along with regular selling can improve total profits even under the situation where there is a cannibalization effect due to some possible switchers to the opaque channels. This rationale of using opaque selling has already been demonstrated in almost all the papers reviewed above by implementing different models under various situations.

Table 2.7 summarized the related research to online opaque selling as well as their model set up information. The followings are the definitions of some of the notations in the table:

REG - refers to regular full information online travel sites (e.g. Expedia.com).

OPQ - refers to posted opaque online travel sites (e.g. Hotwire.com).

BID - refers to opaque bidding online travel sites (e.g. Priceline's Name Your Own Price).

EX - refers to the situation where customer segments are exogenously given (e.g. Brand-loyals and Switchers, Business and Leisures).

EN - refers to the situation where customer segments are endogenously given (e.g. choosing the channel or product by maximizing (or minimizing) their surplus (or cost)).

N/A - refers to the situation where there are no customer segments since there is only one channel considered in the model.

Table 2.7: Related Research to Online Opaque Selling

Reference	Seller	Model Information	
		Channel Structure	Customer Segments
Deneckere and McAfee (1996)	One	Analogous to REG+OPQ	Both EX and EN
Zettelmeyer (2000)	Multiple	Analogous to REG+OPQ	EX
Chiang et al. (2003)	One	Analogous to REG+OPQ	EN
Hann and Terwiesch (2003)	One	Priceline	N/A
Caldentey and Vulcano (2004)	One	Analogous to REG+BID	EN
Fay (2004)	One	Priceline	N/A
Gallego and Phillips (2004)	One	Analogous to REG+OPQ	EN
Gallego et al. (2004)	One	Analogous to REG+OPQ	EN
van Ryzin and Vulcano (2004)	One	Analogous to REG+BID	EX
Coughlan and Soberman (2005)	Two	Analogous to REG+OPQ	EX
Ding et al. (2005)	One	Priceline-Like	N/A
Terwiesch et al. (2005)	One	Priceline-Like	N/A
Etizon et al. (2006)	One	Analogous to REG+BID	EN
Jiang (2007)	One	Analogous to REG+OPQ	EX
Fay (2008a)	One	Priceline-Like	N/A
Fay (2008b)	Two	Analogous to REG+OPQ	EX
Fay and Xie (2008)	One	Analogous to REG+OPQ	EN
Shapiro and Shi (2008)	Multiple	Analogous to REG+OPQ	EX
Wilson and Zhang (2008)	One	Priceline	N/A
Anderson (2009)	One	Priceline	N/A
Cai et al. (2009)	Two	Analogous to REG+BID	EX
Fay (2009)	Two	Analogous to OPQ+BID	EX
Fay and Laran (2009)	One	Priceline-Like	N/A
Wang et al. (2009)	One	Analogous to REG+BID	EX
Jerath et al. (2010)	Two	Analogous to REG+OPQ	EN

## CHAPTER 3

### A CHOICE BASED DYNAMIC PROGRAMMING APPROACH FOR SETTING OPAQUE PRICES

#### 3.1. Introduction

Using opaque products allows firms to reach price sensitive consumers by offering discounts given the uncertainty of the seller while simultaneously selling at higher prices to brand loyal consumers on regular non-opaque channels. While opaque selling is most commonly used in the selling of travel services and pioneered by Priceline.com, as discussed in Fay (2008) it offers opportunities for numerous retailers. The pricing of services (rooms, rental cars, airline seats, etc...) online has dramatically changed how service firms reach customers, with online travel sales now exceeding offline (or traditional sales channels). Initial thoughts about pricing online were very positive as firms had new channels to reach customers enabling increased opportunities for segmentation. Over time service providers have increased efforts to move customers back to company direct distribution channels (company websites and call centers) in an effort to control sales costs and commissions while maintaining direct contact with the customer to facilitate loyalty programs and other marketing efforts. Hotwire and Priceline, unlike other online travel sites such as Expedia, Travelocity and Orbitz, offer customers opaque products (e.g. hotel rooms, flights and rental cars) with aspects of the service provider concealed until the transaction has been completed. For instance a customer purchasing a hotel room through Hotwire can only specify check-in/out dates, a subarea within a city and a star rating. Customers do not know the identity or exact location of their

non-refundable choice property until after purchase. Opaque travel sites offer service providers a convenient channel to segment customers and distribute discounted products without cannibalizing or diluting full priced products. The opaque channels naturally segment customers as regular full price paying customers desiring to stay at the hotel of their choice with full cancelation flexibility are different from those willing to purchase the non-refundable opaque product at the unknown service provider. Therefore service providers need to understand better how to optimally set prices on opaque channels. This question is the focus of our paper.

In this paper we develop a nested logit (NL) model of customer choice using data from transactions occurring at Hotwire.com. The choice model provides insight into the impact of property characteristics that customers can access before purchases (i.e. price, star rating, neighborhood) on the market share. We also estimate a logistic regression model using data from a Washington DC based hotel to determine whether or not a service firm is displayed at Hotwire. These two price dependent models are then combined with dynamic programming to determine optimal prices for the service firm to post at Hotwire.

There is a growing interest in modeling demand using discrete choice models to improve revenue management and pricing. Unlike most choice-based revenue management research which uses an individual firm's sales data for parameter estimation, our data includes market level purchase transaction information (purchases across all service providers, not a single provider) as well as records of the requests that did not induce an actual purchase. We solve a stochastic dynamic program to set optimal choice-based prices for a service provider to post on the intermediary. We characterize the structure of this opti-

mal policy and benchmark optimal fixed price policies against dynamic pricing policies.

In Section 3.2 we briefly summarize the related literature, Section 3.3 describes our data and estimation of the NL model as well as the logistic regression, with Section 3.4 devoted to the development of the dynamic program for setting dynamic and daily fixed prices. A numerical example in Section 3.5 is used to illustrate how pricing policies change with capacity and length of selling horizon. Finally, we provide some conclusions and possible future work in Section 5.5.

## **3.2. Literature Review**

Most of the early research on opaque pricing focusses on Priceline's Name-Your-Own-Price mechanism or similar opaque bidding channels (e.g. Anderson 2009; Fay 2004; Hann and Terwiesch 2003; Terwiesch, Savin and Hann 2005 and Wang et al. 2006). As we focus on the posted price opaque model of Hotwire versus the bidding format of Priceline we refer readers to Anderson (2009) for a review of this research stream. Here we summarize research on posted price opaque mechanisms. Fay (2008) is the first paper to investigate how product opacity affects the market. Fay studies two competing service providers selling products to two types of customers (business and leisure) on both an opaque posted price channel and a traditional distribution channel. Fay shows that opaque selling benefits the monopoly service provider when customers have heterogeneous values for products. Similar to Fay (2008), Jiang (2007) also considers heterogeneous customers and studies how online opaque selling, as a new price dis-

crimination technique, improves profits for service providers. Jiang compares opaque selling and regular selling (selling full-information products), providing insight when to implement opaque selling. Jiang also investigates the impact of opaque selling on social welfare. Shapiro and Shi (2008) focus on providing a rationale for opaque selling. They explain why service providers are willing to distribute products through opaque travel sites such as Priceline and Hotwire and lose the advantage of product differentiation. Jerath, Netessine and Veeraraghavan (2007) compare opaque selling with last-minute direct selling and obtain the conditions under which opaque selling is preferred.

Research on the application of choice models is quite diverse in both the marketing as well as the operations literature. Discrete choice models have been employed to study consumer choice behavior and understand consumer's brand preferences, market structure as well product attributes. Coretjens and Gautschi (1983) provides a general survey of discrete choice models in marketing and a systematic introduction to discrete choice modeling theory. During the past 40 years, the multinomial logit (MNL) model has been the most popular discrete choice model used by marketing researchers (e.g. Guadagni and Little 1983, Krishnamurthi and Raj 1991, Gönül and Srinivasan 1993, Fader and Hardie 1996). MNL models require alternatives to have the same relative probability of being selective independent of the choice set, the so called Independence of Irrelevant Alternatives (IIA) property. The IIA property of the MNL model has given rise to the development of nested models. Dalal and Klein (1988) give several approaches to relax the IIA assumption and the nested logit model is one of them. Since the nested logit model is a natural generalization of the MNL model (McFadden 1981, Ben-Akiva and Lerman 1985) and it can be estimated easily, it has been commonly implemented by marketing researchers.



By adding the customer's decision on the selection of the category, Guadagni and Little (1998) extend the MNL formulation in Guadagni and Little (1983) which only models the brand choice for coffee purchase using an NL model that include both decision components (nested on categories). They demonstrate that the NL model allows a more accurate calculation of sales and a better forecasting of market response to store promotion. They use 32 weeks of coffee purchase panel data collected from four Kansas City stores, which contains information such as the date, the price, the item purchased and the household buying for each coffee purchase. Similarly, our model is based on 6 weeks of booking and shopping data in Washington DC from Hotwire, which includes, for each purchase and search, the date, the property's star rating, neighborhood and price. Because the properties located in the same neighborhood share some characteristics, our NL model is nested on the neighborhood. Guadagni and Little (1998) use a sequential estimation technique (estimate separate levels of the NL tree in sequential order from the bottom to the top), similar to Dubin (1986) and Kannan and Wright (1991). However, we implement the full maximum likelihood estimation i.e. estimating all the levels of the NL simultaneously-as it can yield statistically consistent and asymptotically efficient estimates of the parameters that sequential estimation cannot (Ben-Akiva and Lerman 1985).

Over the past decade, choice based models have been introduced in the revenue management literature where both price and inventory decisions need to be made. Andersson (1998) using data from Scandinavian Airlines System uses a MNL model to estimate passenger preferences among buying up, being recaptured and deviating to another airline. Talluri and van Ryzin (2004b) study consumer choice behavior among multiple fare classes on a single-leg flight using a MNL model. They formulate the optimal seat allocation policy as a dynamic

program (DP). This formulation is extended by Liu and van Ryzin (2008) to a network case, in which their DP problem is approximated by a deterministic linear programming problem. Similarly, Gallego et al. (2004) also use a deterministic approximation to solve a network RM problem where newly defined *flexible products* are chosen endogenously by customers. Zhang and Adelman (2007) also consider a network revenue management problem where consumer choice behavior is explicitly modeled by a MNL model. They claim that they provide a better approximation to the resulting DP model of Liu and van Ryzin.

Our paper is similar to Talluri and van Ryzin (2004b), Liu and van Ryzin (2008), Zhang and Adelman (2007) in that we all use a discrete choice model to characterize consumer choice behavior and formulate the revenue management problem as a dynamic programming problem. However, we solve the resulting stochastic DP problem versus assuming a priori optimal policy. This difference is due to the fact that we use price as the decision variable in the DP instead of deciding which subsets of fare classes to offer to customers. To our knowledge, this paper is the first attempt in the literature to characterize customer choice behavior by using a NL model versus a MNL model.

Zhang and Cooper (2005) study a seat allocation problem when customers dynamically choose among multiple parallel flights. Similar to our model setting, they also assume that there is at most one arrival in each period and the arrivals follow a non-homogeneous Poisson process. They do not model customer choice behavior in a discrete choice framework, similar to this paper, they assume that customers choose products from different inventory resources. In contrast, other papers from the literature examine customer choice between products using a common pool of resources.

More recently Vulcano et al. (2010) apply the simulation based optimization approach used in van Ryzin and Vulcano (2008) to estimate optimal controls where passenger demand is estimated use choice based models approximated with the expectation-maximization algorithm owing to their use of censored data. Gallego et al. (2009) use a choice based model in conjunction with EMSR heuristic, see Belobaba (1987), to set leg level inventory controls. Similar to Andersson (1998) and Algers and Besser (2001), Vulcano et al. (2010) perform an empirical choice model study of estimation and optimization on actual airline data from a major U.S. airline. Both the simulated and real data in all the work above are using sales transactions from one service provider. In contrast, our data contains complete market level booking and shopping records, which allows us to use the traditional maximum likelihood method (MLE) estimation. This unique data also allows us to truly and better characterize the price competition, since the competitors' prices are included in our data (not in one company's sales data) and so were captured in the choice probability resulting from the choice model. As far as we know this work is also the first choice-based revenue management research applied to the hotel setting, whereas almost all prior literature focused on airline revenue management systems.

### **3.3. Demand Models for Opaque Products**

In the following section we briefly describe our data set and outline the application of a nested logit choice model to this data set.

### 3.3.1 Data

Our data consists of daily data on all hotel booking requests and purchase transactions made through Hotwire in Washington DC over an arrival period of 6 weeks. As the first request was made 321 days before arrival (DBA), our data set extends beyond one year. Each data record consists of the date the request was made (date consumer shopped online), the check-in and check-out dates, and characteristics of the properties such as price, star rating and neighborhood within DC. The data also contains information on whether a request led to a purchase transaction.

There are 195,226 requests in total in the data set and 7,509 of them lead to a reservation. Looking at 7,505 reservations (we drop 4 of the 7,509 as these are the only 4 reservations for 4.5 star properties) we get a feel for what sort of properties customers are booking and where they are located within DC. Table 3.1 displays the percent of reservations or market share on Hotwire by hotel star level. Table 3.2 shows a similar market share summary by geographic subarea (neighborhood) within Washington, DC. While requests were made as early as 321 DBA, over two thirds of reservations are made within the last 10 DBA.

From Tables 3.1 and 3.2 we see each star and area receiving some fraction of reservations with stars 3, 3.5 and 4 dominating and Dupont Circle-Embassy Row the most popular area. Figure 3.1 illustrates some of the price tradeoffs customers are making when selecting hotel star levels. In Figure 3.1 the horizontal axis is a 4 star property's price relative to the 3.5 star property's prices in the same neighborhood. A relative price of 100% means the 4 and 3.5 star property were priced the same, with 80% indicating the 4 star was 20% cheaper than the 3.5 star. The bars, the primary vertical axis, represent 4 star purchases at each

Table 3.1: Percentage of reservations by hotel star level, Washington DC

Star rating	Percentage of booking (%)
1	0.1
2	5.2
2.5	4.2
3	18.9
3.5	15
4	56.6

Table 3.2: Percentage of reservations by Washington, DC subarea

Neighborhood	Percentage of booking (%)
Alexandria-Old Town	12.3
Arlington	3.5
Bethesda-Silver Spring	2.4
Chantilly-Dulles Intl Airport IAD South	4.3
Crystal City-Reagan National Airport DCA	2.3
Dupont Circle-Embassy Row	45.5
Georgetown	1.8
Kennedy Center-GW University	0.6
Reston-Sterling-Dulles Intl Airport IAD North	2.5
South of the Mall	0.8
The White House-Capitol Hill	24

price point as a percentage of all 4 star purchases. As shown in the graph, the majority of the purchases are made at properties with relative prices of 90% or less. The solid line (the secondary vertical axis) corresponds to the market share

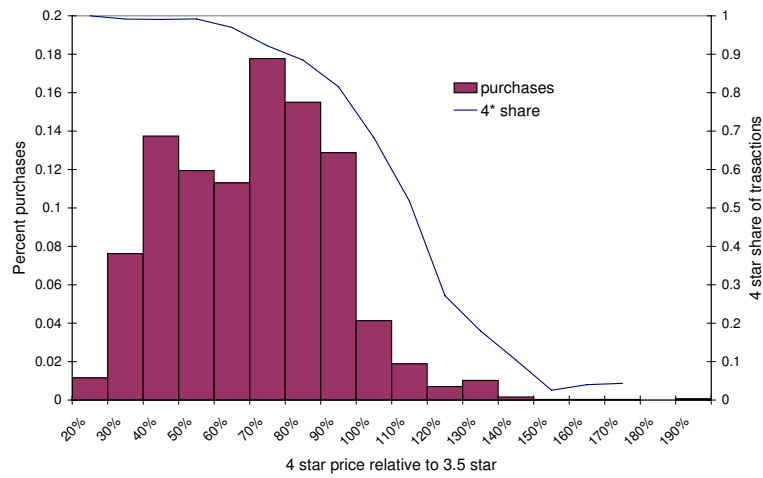


Figure 3.1: Distribution of reservations and market share star level price ratios

of 4 star properties at each discount. We see market share decrease with relative price increases. We note that even when the 4 star property is 30% cheaper than a 3.5 star (relative price of 70%), there are still sales made to other star qualities as market share is less than 1. Figure 3.1 indicates that while customers are making price tradeoffs (price is a primary driver of the purchase decision) there must be other factors motivating purchase choice - motivating our use of choice models for estimating market share or purchase probability. An interesting observation is the existence of price inversion where hotels of higher star rating are priced cheaper than lower rated hotels. Figure 3.1 shows that 4 star properties tend to have very high market share under price inversion, but that drastic price inversion does not occur that often as indicated by the height of the bars.

### 3.3.2 The nested logit model and estimation

A discrete choice model is designed to explain how an individual decision maker makes a choice among a feasible set of alternatives. In this case, the decision makers are the consumers who make requests and book a hotel located in Washington D.C. through Hotwire.com. We represent the consumer choice behavior using the nested logit model. Although the MNL model is widely used in modeling choice behavior, it requires alternatives in the choice set to have the Independence of Irrelevant Alternatives (IIA) property. The IIA property requires that changes in the choice set do not affect the ratio of choice probabilities of any two alternatives. Under this property, all alternatives should have the same measure of similarity. The IIA property is often violated when some alternatives share attributes (are similar), but some do not (are dissimilar). The nested logit model is an extension to the MNL model which was introduced to relax the IIA assumption by grouping similar alternatives.

Specifically, consumer  $n$  is faced with a decision of booking a property at a star level  $j$  within a neighborhood  $i$  in Washington D.C. The universal choice set of alternatives denoted by  $C$  consists of all the properties identified by an  $(i, j)$  pair (neighborhood, star). Each consumer  $n$  has a corresponding choice set  $C_n \subset C$ . We partition the alternatives into 11 groups by neighborhood. In other words, the properties in each group are in the same neighborhood. We need to construct a nested logit model to avoid the violation of the IIA property as comparing a 3 star to a 4 star hotel in the same neighborhood is a different tradeoff compared to choosing between a 3 star in one neighborhood over a 3 star hotel in another neighborhood. In our two-level NL model, neighborhood is the first-level feature  $i$  and star is the second-level feature  $j$ . The random

utility that consumer  $n$  books a property of star quality  $j$  within neighborhood  $i$  is given by

$$\begin{aligned} U_{nij} &= \beta' \mathbf{x}_{nij} + \varepsilon_{ni} + \varepsilon_{nij} \\ &= \beta_p x_{nijp} + \beta_{s1} x_{njs1} + \beta_{s2} x_{njs2} + \beta_{s3} x_{njs3} + \beta_{s4} x_{njs4} + \beta_{s5} x_{njs5} + \varepsilon_{ni} + \varepsilon_{nij} \end{aligned} \quad (3.1)$$

where  $\mathbf{x}_{nij} = (x_{nijp}, x_{njs1}, x_{njs2}, x_{njs3}, x_{njs4}, x_{njs5})'$  is the vector of attributes of a property at star level  $j$  in a neighborhood  $i$ .  $x_{nijp}$  represents the price of alternative  $(i, j)$  and  $x_{njsi}$  are indicator variables for 5 star qualities (1, 2, 2.5, 3, and 3.5 (stars) with the reference a 4 star hotel.  $\beta = (\beta_p, \beta_{s1}, \beta_{s2}, \beta_{s3}, \beta_{s4}, \beta_{s5})'$  is the parameter vector that captures the consumer's preferences for those attributes and it needs to be estimated.

$\varepsilon_{nij}$  is the random error term corresponding to the alternative  $(i, j) \in C_n$ , which follows a Gumbel distribution.  $\varepsilon_{ni}$  is the random error term corresponding to neighborhood  $i$  and is distributed such that  $\max_{j \in J_{ni}} U_{nij}$  is Gumbel distributed with scale parameter  $\mu^I$ , commonly known as the dissimilarity coefficient.  $J_{ni}$  is defined as the set of all star qualities available for consumer  $n$  if he or she chose neighborhood  $i$  at the first level.

The probability that consumer  $n$  chooses alternative  $(i, j) \in C_n$  is given by:

$$\begin{aligned} P_n(i, j) &= Pr[U_{ij} \geq U_{i'j'} \text{ for all } (i', j') \in C_n] \\ &= P_n(i) \cdot P_n(j|i) \\ &= \frac{e^{V_i' \mu^I}}{\sum_{i' \in I_n} e^{V_{i'}' \mu^I}} \cdot \frac{e^{\beta' \mathbf{x}_{nij}}}{\sum_{j' \in J_{ni}} e^{\beta' \mathbf{x}_{nij'}}} \end{aligned} \quad (3.2)$$

where,

$$V_i' = \ln \sum_{j \in J_{ni}} e^{\beta' \mathbf{x}_{nij}}, \quad (3.3)$$



is called the inclusive value and represents the scaled effect of the utilities in the second-level on the first-level's utility.  $I_n$  is the set of all neighborhoods in  $C_n$ .

Table 3.3: Nested logit parameter estimates

Parameter	Parameter Estimate	Standard Error	P-value
$\beta_p$	-0.06941	0.0016	0.0000
$\beta_{s1}$	-7.7288	0.3784	0.0000
$\beta_{s2}$	-6.2962	0.1259	0.0000
$\beta_{s3}$	-5.0331	0.1121	0.0000
$\beta_{s4}$	-3.1586	0.0704	0.0000
$\beta_{s5}$	-1.9744	0.0592	0.0000
$\beta_{DBA}$	-0.0153	0.0037	0.0000
1st-level Feature Neighborhood	Dissimilarity Coefficient	Standard Error	
Alexandria-Old	0.6874	0.0187	0.0000
Arlington	0.7376	0.0202	0.0000
Bethesda-Silver Spring	0.7425	0.0217	0.0000
Chantilly-Dulles Intl Airport IAD South	0.9049	0.0243	0.0000
Crystal City-Reagan National Airport DCA	0.6674	0.0183	0.0000
Dupont Circle-Embassy Row	0.4678	0.0141	0.0000
Georgetown	0.5832	0.061	0.0000
Kennedy Center-GW University	0.7361	0.0249	0.0000
Reston-Sterling-Dulles Intl Airport IAD North	0.9700	0.0264	0.0000
South of the Mall	0.6068	0.0178	0.0000
The White House-Capitol Hill	0.4463	0.0129	0.0000

We estimate the parameters  $\beta_p, \beta_{si}$  for  $i = 1, 2, \dots, 5$  and obtain the choice probabilities by applying the maximum likelihood method using NLOGIT (Greene, 2007). Table 3.3 summarizes parameter estimates. As shown in Table 3.3 all the estimates of the parameters are significant and have the expected sign. The parameters for price are negative, implying that consumers prefer properties with lower prices. The parameters for star rating's indicator variables are all negative

with  $\beta_{s1} < \beta_{s2} < \dots < \beta_{s5}$  indicating customers prefer higher quality hotels. The dissimilarity coefficients are all significantly different from zero and lie in the interval  $[0, 1]$  consistent with random utility maximization.

Table 3.4: Nested logit model fit diagnostics

LL	-10979.11
LL*	-29503.89
McFadden Pseudo R-squared	.6278757

Table 5.8 summarizes model fit characteristics. The McFadden Pseudo R-squared is an indicator of the overall model fit, which is

$$1 - \frac{LL}{LL^*} = 1 - \frac{-10979.11}{-29503.89} = 0.628,$$

where  $LL$  is the log-likelihood of the test model and  $LL^*$  is the log-likelihood for a reference model which is estimated with constant only, i.e. the market shares predicted by the model are what are in the data. The NL model correctly classified 68.9% of choices with the modal category being a 4 star property in Dupont Circle (39 %) of purchases resulting in a proportional reduction in classification error of  $\frac{.689-.39}{1-.39} = 49.0\%$ .

In addition to modeling consumer choices, the NL model can be used to determine consumer price sensitivity or elasticity as well as the monetary value of each star rating. Table 3.5 summarizes the average (across the 11 neighborhoods in DC) of own and cross price elasticities. Price elasticities from the NL model are the percent change in choice probability given a percent change in price. In Table 3.5 the diagonal (negative values) are the own price elasticities and the off diagonal (positive values) are the cross price elasticities. For example in the first row (1 star hotel) the choice probability decreases 1.36 % for each 1 % increase in

the 1 star hotel's price, whereas the 2 star property's choice probability increases 0.02% with this same 1 percent increase. The higher star value properties tend to be more elastic with the 4 star's price having the largest impact on the other star's choice probabilities (largest cross price effects). The cells which are empty indicate that these two types of properties were never simultaneously listed, e.g. a 4 star property was never displayed with a 1 star property.

Table 3.5: Own and cross price elasticities by star level

Star	Star					
	1	2	2.5	3	3.5	4
1	-1.36	0.02	0.01	0.02	0.02	
2	0.13	-2.86	0.11	0.26	0.16	0.22
2.5	0.24	0.12	-4.0	0.2	0.2	0.25
3	0.41	0.75	0.92	-4.83	1.03	0.9
3.5	0.53	0.67	0.53	0.64	-4.67	0.84
4		2.4	2.43	2.05	2.4	-4.55

The choice model can also be used to quantify the financial impact of different star ratings. Focusing on the 7505 reservations in the data set, for each purchased property we can use the parameters from Table 3.3 to calculate the utility of the selected property. Then decrease the star rating of the selected property by one level (a 4 star becomes a 3.5 star) then we calculate the change in price needed such that this decreased quality property has the same utility as the higher quality property. Table 3.6 summaries these impacts, e.g. if a 2 star was to change to a 1 star it would need to drop its price by \$20.58 or 32% to have the same utility (and same market share or choice probability). The higher star ratings tend to have a higher marginal value.

Table 3.6: Financial value of star rating

Star	Price Change (\$)	Percentage Price Change (%)
2	20.58	32
2.5	18.2	30
3	27	35
3.5	17.06	18
4	28.44	30

The NL estimates the probability of making a sale assuming the firm is displayed in the choice set, in the following section we estimate a logistic regression model that estimates the probability a firm is displayed.

### 3.3.3 Probability of Display

Figure 3.2 shows a subset of the information a hotel (or prospective consumer) would have available about its market for any future arrival date. The figure shows a set of prices for hotels by star class and location within Washington DC. Figure 3.3 shows a map of the 11 neighborhoods within DC with each of these 11 neighborhoods potentially having one property at each star level displayed similar to those in Figure 3.2. Not all service providers releasing inventory to Hotwire will be displayed in Figure 3.2. For instance, there are thirteen 3.5 star properties in Dupont Circle in Washington DC, all may release inventory to Hotwire, but given the opaque structure of Hotwire only one property is displayed, i.e. gets an opportunity at a sale on Hotwire. Hotwire provides a daily report to each property which can be used to determine its likelihood of being displayed in the future. A sample daily report is displayed in Figure 3.4.

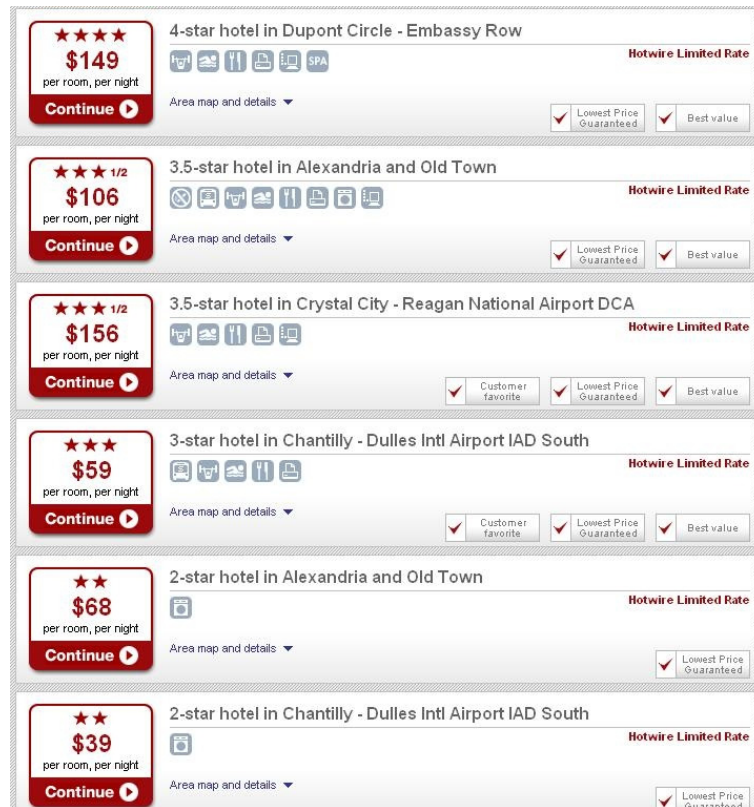


Figure 3.2: Sample hotel listing on Hotwire.com

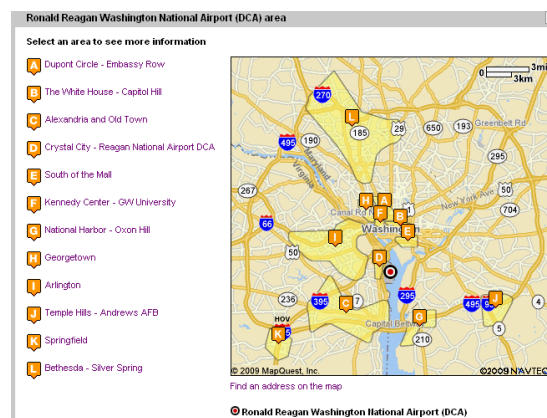


Figure 3.3: DC area neighborhoods on Hotwire.com

The report contains summaries on price (firm and competitors), the number of requests made as well as the number of times the firm was displayed on that day.

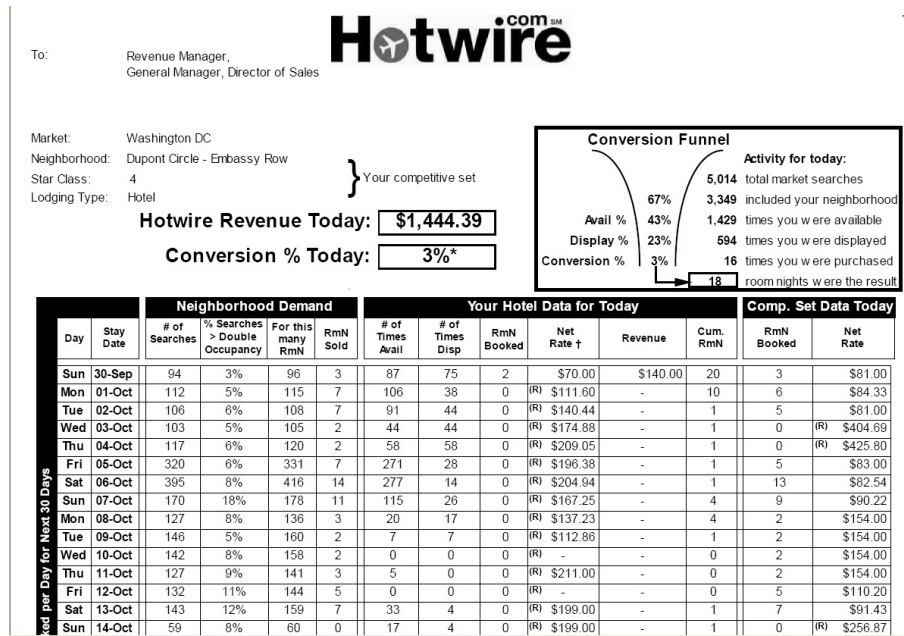


Figure 3.4: Sample Hotwire.com report

Figure 3.4 summarizes all Hotwire activity made the previous day for all future arrival dates, with each row in the report representing a future arrival date. The data in the middle block of columns (Your Hotel Data for Today) indicates for how many consumer searches the hotel had available inventory (# of Times Avail) as well as the number of times it was displayed (# of Times Disp) and the prices it had posted to Hotwire (Net rate). For example, for an arrival on Sunday, September 30th there were 94 customer searches, of these 94 the hotel had rooms available 87 times, the hotel was displayed 75 times and 2 reservations were made.

We use this data from 6 weeks of daily reports to estimate the probability that

a hotel will be displayed on as a function of price using logistic regression. Let  $P_d(p(t))$  represent the probability of display for a particular arrival date posting a price  $p(t)$  at  $t$  days prior to arrival, then

$$P_d(p(t)) = \frac{1}{1 + \exp(g(p(t)))} \quad (3.4)$$

where  $g(p(t))$  is a linear function of  $p(t)$ .

We transform price into relative price, a firm's price divided by the displayed comparable price. The displayed price is the price of the currently displayed hotel of the same star level in the same neighborhood (e.g. 3 star hotel in Dupont Circle). As prices change the closer you book relative to the stay date we also include DBA, i.e. days before arrival (stay date-search date) as an independent variable as well as an indicator variable for weekend versus weekday as weekend prices tend to be lower than weekday.

The model specification that we estimated from our data is,

$$g(p(t)) = -\alpha_0 - \alpha_1 \frac{p(t)}{p_{competitor}(t)} - \alpha_2 DBA - \alpha_3 weekend$$

Therefore  $P_d(p(t)) =$

$$\frac{1}{1 + \exp(-\alpha_0 - \alpha_1 \frac{p(t)}{p_{competitor}(t)} - \alpha_2 DBA - \alpha_3 weekend)}. \quad (3.5)$$

Table 3.7 summarizes parameter estimates. Price ratio is the ratio of our price to the comparable price. Weekend is an indicator, a 1 if the date we are pricing is a weekend, 0 otherwise.

Figure 3.5 shows a set of sample results using (3.5) and the estimates in Table 3.7 for a firm posting prices for a weekend ( $Weekend = 1$ ) arrival day ( $DBA = 0$ ) with competitors posting a price of \$150.

Table 3.7: Logistic regression coefficients for probability of display model

Parameter	Coefficient Estimate
Constant	0.593*
Price Ratio	-1.684*
<i>DBA</i>	0.110*
Weekend	-0.892*

\*significant at the 0.001 level

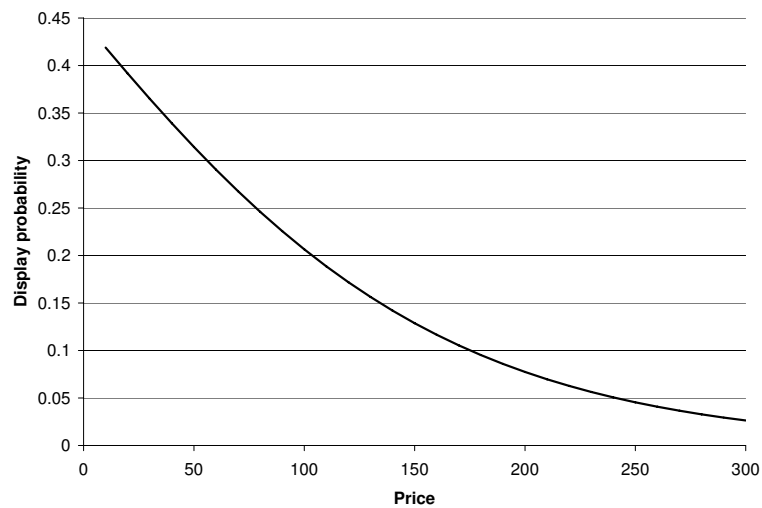


Figure 3.5: Display probabilities from logistic regression for prices posted on arrival day for a weekend arrival with a \$150 competitor rate

### 3.3.4 Fraction of customers buying - making a sale on Hotwire

Section 5.8 and 3.3.3 outlined the estimated nested logit model and logistic regression models. The nested logit model determines the probability a firm



makes a sale given a sale is made and the firm is displayed. The logistic regression estimates the probability a firm is displayed. Figure 3.6 shows a sample graph of the average number of requests made, referred to as looks, and the percentage of these requests making a reservation (book-to-look ratio).

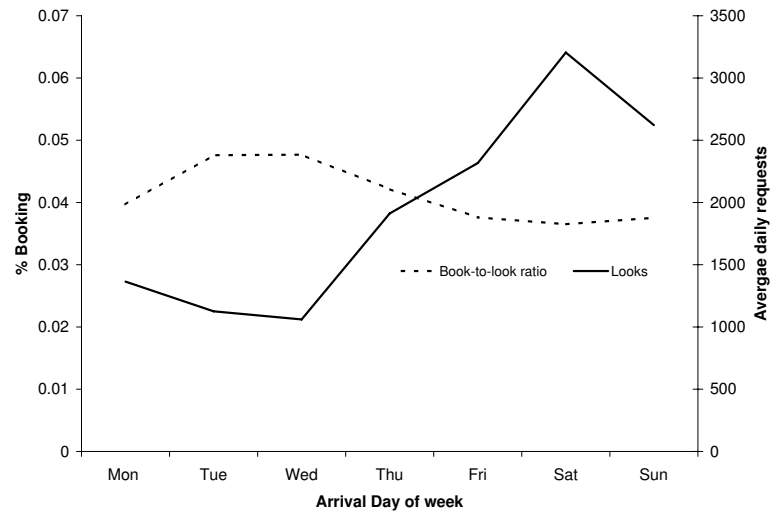


Figure 3.6: Average number of daily request (looks) and percentage making a reservation (book-to-look ratio)

Figure 3.6, the nested logit and logistic regression models can be used together to develop a price dependent customer arrival process. We outline two forms of the arrival process in the following section, one assuming dynamic prices, another for daily fixed prices.

### 3.4. Pricing Model Development

We determine optimal prices under two separate frameworks, later comparing prices and expected revenues. In Section 3.4.1 we assume dynamic pricing over the entire selling horizon. In Section 3.4.2 we assume the firm changes prices each day but that prices remain fixed throughout the day, for example a hotel accepting room reservations for a stay date 3 days from today would post at most 4 different prices for that room (potentially a different price on each of the arrival date and each day prior to arrival). We evaluate optimal prices under these two frameworks. Dynamic pricing provides increased revenues and allows a characterization of the optimal policy. By contrast firms traditionally use daily fixed prices largely as a result of reservation systems limitations.

#### 3.4.1 Dynamic Pricing

Customers are assumed to arrive at Hotwire following a nonhomogeneous Poisson process with an arrival rate  $\lambda_t$ ,  $t$  days prior to arrival. We subdivide each day prior to arrival into  $\eta_t$  decision periods of equal lengths such that there is at most one arrival request in each period. More specifically, the value of  $\eta_t$  should be such that the length of each decision period is small enough to have negligible probability  $\varepsilon$  of more than one arrival. The arrivals for each decision period follow a Poisson process with rate  $\lambda_t/\eta_t$  and the probability of  $x$  arrivals in a decision period  $m$  is given by

$$P^m(x) = \frac{(\lambda_t/\eta_t)^x \exp^{-(\lambda_t/\eta_t)}}{x!}, \quad (3.6)$$

where  $x = 0, 1, 2, \dots$  and  $1 - P^m(0) - P^m(1) \leq \varepsilon$ . For convenience, we denote the arrival probability of one request in decision period  $m$  by  $P^m$  as,

$$P^m = (\lambda_t / \eta_t) \exp^{(\lambda_t / \eta_t)}. \quad (3.7)$$

As not every arriving customer chooses to book on Hotwire, let  $P_b(m)$  be the probability that an arrival makes a reservation (the book-to-look ratio in Figure 3.6) in decision period  $m$ . Consider a property releasing  $r$  rooms over the next  $m$  periods at price  $p(m, r)$  who is displayed on Hotwire with choice probability  $P_c$  given by the nested logit model (3.2).

The probability, denoted  $P_s$ , that a randomly selected customer will be willing to book a room is a function of its price  $p(m, r)$  and given by

$$P_s(p(m, r)) = P_d(p(m, r))P_b(m)P_c(p(m, r)) \quad (3.8)$$

The probability,  $f_{p(m, r)}$ , of making a sale at the price  $p(m, r)$  becomes

$$\begin{aligned} f_{p(m, r)} &= P_s(p(m, r))P^m \\ &= P_d(p(m, r))P_b(m)P_c(p(m, r))P^m \end{aligned} \quad (3.9)$$

where  $P_d(p(m, r))$  is given by (3.4),  $P_c(p(m, r))$  is given by (3.2), and  $P^m$  is given by (3.7) above. We can use this price dependent probability of making a sale to determine the optimal price to post via dynamic programming.

Let  $V^*(m, r)$  denote the optimal total expected revenue, resulting from price  $p^*(m, r)$ , that can be generated from the remaining  $m$  periods given  $r$  remaining rooms, where  $m = \{0, 1, 2, \dots, M\}$  and  $r = \{0, 1, 2, \dots\}$ . If a sale is made in period  $m$ , then  $V^*(m, r)$  is equal to  $p(m, r) + V^*(m - 1, r - 1)$ . If there is no sale in that

period, then the optimal total expected revenue is given by  $V^*(m-1, r)$ .  $V^*(m, r)$  is characterized by the following recursion,

$$\begin{aligned} V^*(m, r) &= \max_{p(m, r)} \{ [p(m, r) + V^*(m-1, r-1)]f_{p(m, r)} + V^*(m-1, r)(1 - f_{p(m, r)}) \} \\ &= V^*(m-1, r) + \max_{p(m, r)} \{ [p(m, r) - \delta(m-1, r)]f_{p(m, r)} \} \end{aligned} \quad (3.10)$$

where  $\delta(m, r) = V^*(m, r) - V^*(m, r-1)$  is the expected marginal value of a room in decision period  $m$  given  $r$  rooms. The boundary conditions are  $V^*(0, \cdot) = 0$ ,  $V^*(\cdot, 0) = 0$ . We next describe characteristics of the optimal pricing policy.

### Optimal Dynamic Pricing Policy

The optimal dynamic pricing policy is described by Theorem 1. Lemmas 1-4 are used in the proof of Theorem 1. Theorem 1 states that the optimal pricing policy is decreasing in capacity and increasing in time. Lemmas 1,2 and 4 are similar in structure to Lemmas 4,5 and 3 respectively in Talluri and van Ryzin (2004b). We use price as our decision variable whereas Talluri and van Ryzin (2004b) characterize an optimal nested inventory allocation policy. The Corollary and Lemma 3 are unique developments with the corollary naturally following Lemmas 1 and 2, and Lemma 4 dependent upon Lemma 3.

**Lemma 1.** The expected marginal value is decreasing in the remaining capacity, i.e.  $\delta(m, r) \leq \delta(m, r-1)$ .

*Proof.* By induction on  $m$ . For  $m = 0$  the boundary condition  $V^*(0, \cdot) = 0$  gives us  $\delta(0, r) = \delta(0, r-1) = 0$ , the lemma is trivially true for this case.

We assume it is true for period  $m-1$ .  $p^*(m, r)$  denotes the optimal solution

for (3.10). From (3.10) above, we get

$$V^*(m, r-1) = V^*(m-1, r-1) + \max_{p(m, r-1)} \{ [p(m, r-1) - \delta(m-1, r-1)] f_{p(m, r-1)} \} \quad (3.11)$$

Then (3.10)-(3.11) gives

$$\begin{aligned} \delta(m, r) - \delta(m-1, r) &= \max_{p(m, r)} \{ [p(m, r) - \delta(m-1, r)] f_{p(m, r)} \} \\ &\quad - \max_{p(m, r-1)} \{ [p(m, r-1) - \delta(m-1, r-1)] f_{p(m, r-1)} \} \\ &= [p^*(m, r) - \delta(m-1, r)] f_{p^*(m, r)} \\ &\quad - [p^*(m, r-1) - \delta(m-1, r-1)] f_{p^*(m, r-1)} \end{aligned} \quad (3.12)$$

From (3.12), we get

$$\begin{aligned} \delta(m, r-1) - \delta(m-1, r-1) &= [p^*(m, r-1) - \delta(m-1, r-1)] f_{p^*(m, r-1)} \\ &\quad - [p^*(m, r-2) - \delta(m-1, r-2)] f_{p^*(m, r-2)} \end{aligned} \quad (3.13)$$

Similarly (3.12)-(3.13)

$$\begin{aligned} \delta(m, r) - \delta(m, r-1) &= \delta(m-1, r) - \delta(m-1, r-1) \\ &\quad + [p^*(m, r) - \delta(m-1, r)] f_{p^*(m, r)} \\ &\quad - [p^*(m, r-1) - \delta(m-1, r-1)] f_{p^*(m, r-1)} \\ &\quad - [p^*(m, r-1) - \delta(m-1, r-1)] f_{p^*(m, r-1)} \\ &\quad + [p^*(m, r-2) - \delta(m-1, r-2)] f_{p^*(m, r-2)} \end{aligned} \quad (3.14)$$

The optimality of the price  $p^*(m, \cdot)$  implies the following inequalities:

$$[p^*(m, r-1) - \delta(m-1, r-1)] f_{p^*(m, r-1)} \geq [p^*(m, r) - \delta(m-1, r-1)] f_{p^*(m, r)} \quad (3.15)$$

$$[p^*(m, r-1) - \delta(m-1, r-1)] f_{p^*(m, r-1)} \geq [p^*(m, r-2) - \delta(m-1, r-1)] f_{p^*(m, r-2)} \quad (3.16)$$

Substituting (3.15) and (3.16) into (3.14) we obtain

$$\begin{aligned}
\delta(m, r) - \delta(m, r - 1) &\leq \delta(m - 1, r) - \delta(m - 1, r - 1) \\
&\quad + [p^*(m, r) - \delta(m - 1, r)]f_{p^*(m, r)} \\
&\quad - [p^*(m, r) - \delta(m - 1, r - 1)]f_{p^*(m, r)} \\
&\quad - [p^*(m, r - 2) - \delta(m - 1, r - 1)]f_{p^*(m, r - 2)} \\
&\quad + [p^*(m, r - 2) - \delta(m - 1, r - 2)]f_{p^*(m, r - 2)} \quad (3.17)
\end{aligned}$$

simplifying,

$$\begin{aligned}
\delta(m, r) - \delta(m, r - 1) &\leq [\delta(m - 1, r) - \delta(m - 1, r - 1)][1 - f_{p^*(m, r)}] \\
&\quad + [\delta(m - 1, r - 1) - \delta(m - 1, r - 2)]f_{p^*(m, r - 2)} \quad (3.18)
\end{aligned}$$

By induction, we know that  $\delta(m - 1, r) - \delta(m - 1, r - 1) \leq 0$  and  $\delta(m - 1, r - 1) - \delta(m - 1, r - 2) \leq 0$ . Therefore, we can conclude that  $\delta(m, r) - \delta(m - 1, r) \leq 0$ .

**Lemma 2.** The marginal value is increasing in remaining time, i.e.  $\delta(m, r) \geq \delta(m - 1, r)$ .

*Proof.* Recall from equation (3.12), we have

$$\begin{aligned}
\delta(m, r) - \delta(m - 1, r) &= \max_{p(m, r)} \{ [p(m, r) - \delta(m - 1, r)]f_{p(m, r)} \} \\
&\quad - \max_{p(m, r - 1)} \{ [p(m, r - 1) - \delta(m - 1, r - 1)]f_{p(m, r - 1)} \} \quad (3.19)
\end{aligned}$$

As proved in Lemma 1,  $\delta(m - 1, r) \leq \delta(m - 1, r - 1)$ . Therefore, the following inequality is always true for any value of the price  $p(m, r)$  for all  $m$  and  $r$ ,

$$[p(\cdot, \cdot) - \delta(m - 1, r)]f_{p(\cdot, \cdot)} \geq [p(\cdot, \cdot) - \delta(m - 1, r - 1)]f_{p(\cdot, \cdot)}.$$

Thus,

$$\max_{p(m, r)} \{ [p(m, r) - \delta(m - 1, r)]f_{p(m, r)} \} \geq \max_{p(m, r - 1)} \{ [p(m, r - 1) - \delta(m - 1, r - 1)]f_{p(m, r - 1)} \} \quad (3.20)$$

which implies  $\delta(m, r) \geq \delta(m - 1, r)$ .

**Corollary.** The optimal total expected revenue  $V^*(m, r)$  is increasing and concave in both the number of remaining periods  $m$  and rooms  $r$ .

*Proof.*  $V^*(m, r)$  increases with  $m$  and  $r$  is obvious and straightforward, so we only focus on the concavity of the expected revenue. From Lemma 1 we know  $\delta(m, r) \leq \delta(m, r - 1)$ , which implies

$$V^*(m, r) - V^*(m, r - 1) \leq V^*(m, r - 1) - V^*(m, r - 2),$$

Hence,  $V^*(m, r)$  is concave in  $m$ .

Also,

$$V^*(m, r) - V^*(m - 1, r) = [p^*(m, r) - \delta(m - 1, r)]f_{p^*(m, r)},$$

and

$$V^*(m - 1, r) - V^*(m - 2, r) = [p^*(m - 1, r) - \delta(m - 2, r)]f_{p^*(m - 1, r)}$$

and Lemma 2 gives that  $\delta(m - 1, r) \geq \delta(m - 2, r)$ . Therefore, using the same argument as for proving (3.20) we obtain

$$V^*(m, r) - V^*(m - 1, r) \leq V^*(m - 1, r) - V^*(m - 2, r).$$

This means  $V^*(m, r)$  is concave in  $r$  as well.

Lemma 4 shows that the optimal price in this period is positively related to the marginal value of the previous period. The following lemma is required in the development of Lemma 4.

**Lemma 3.** Assume the property's display probability given by (3.5) and choice probability given by (3.2) are decreasing in price  $p(m, r)$ . Then, the probability of making a sale  $f_{p(m, r)}$  also decreases in price  $p(m, r)$ .

*Proof.* From the expression of  $f_{p(m,r)}$  in (3.9), we have

$$f_{p(m,r)} = P_d(p(m,r))P_b(m)P_c(p(m,r))P^m, \quad (3.21)$$

It is straightforward to see that given  $P_b(m)$  and  $P^m$  do not depend on price  $p(m,r)$  as well as both  $P_d(p(m,r))$  and  $P_c(p(m,r))$  are decreasing in  $p(m,r)$ , one can conclude that  $f_{p(m,r)}$  also decreases in price  $p(m,r)$ .

**Lemma 4.** The optimal price  $p^*(m,r)$  increases with the marginal value  $\delta(m-1,r)$ .

*Proof.* We prove this statement by using contradiction. For ease of notation, set  $\delta = \delta(m-1,r)$  and  $p^* = p^*(m,r)$ . Assume  $\delta_1$  and  $\delta_2$  are two marginal values that satisfy  $\delta_1 > \delta_2 \geq 0$ .  $p_1^*$  and  $p_2^*$  are the optimal prices that maximize the term  $[p(m,r) - \delta(m-1,r)]f_{p(m,r)}$  in (3.10) when  $\delta(m-1,r)$  equals  $\delta_1$  and  $\delta_2$  respectively.  $f_{p_i^*}, i = 1, 2$  is the corresponding probability of making a sale at optimal prices  $p_i^*$ .

Now suppose  $p_1^* < p_2^*$ , we attempt to get a contradiction later.

From Lemma 3, if  $p_1^* < p_2^*$  then  $f_{p_1^*} > f_{p_2^*}$ .

On the other hand, the optimality of  $p_1^*$  implies

$$p_1^* f_{p_1^*} - \delta_1 f_{p_1^*} \geq p_2^* f_{p_2^*} - \delta_1 f_{p_2^*}$$

then

$$\begin{aligned} p_1^* f_{p_1^*} - p_2^* f_{p_2^*} &\geq [f_{p_1^*} - f_{p_2^*}] \delta_1 \\ &\geq [f_{p_1^*} - f_{p_2^*}] \delta_2 \end{aligned} \quad (3.22)$$



Then  $p_1^* f_{p_1^*} - \delta_2 f_{p_1^*} \geq p_2^* f_{p_2^*} - \delta_2 f_{p_2^*}$ . This inequality contradicts the optimality of  $p_2^*$ . Therefore, we must have  $p_1^* > p_2^*$  if  $\delta_1 > \delta_2$ .

By combining Lemmas 1, 2, and 4, we obtain the next theorem that characterizes the optimal pricing policy.

**Theorem 1.** Consider a property that releases  $r$  rooms over the next  $m$  periods at price  $p(m, r)$ . Assume the property's display probability given by (3.5) and choice probability given by (3.2) are decreasing in price  $p(m, r)$ . Then, the optimal pricing policy for (3.10),  $p^*(m, r)$ , is decreasing in the remaining number of rooms  $r$  and increasing in the remaining selling periods  $m$ .

We next show an illustration using our data for which both  $P_d(p(m, r))$  and  $P_c(p(m, r))$  are decreasing in  $p(m, r)$ . For ease of notation, we simply denote  $p(m, r)$  by  $p$ .

Recall from (3.5) that the display probability  $P_d(p)$  is given by

$$P_d(p) = \frac{1}{1 + \exp(-\alpha_0 - \alpha_1 \frac{p}{p_{competitor}} - \alpha_2 DBA - \alpha_3 weekend)}$$

Since the estimate for the price ratio's coefficient  $\alpha_1$  is negative, it is straightforward to see that the display probability  $P_d(p)$  decreases in price  $p$ .

Recall from expression (3.2), the choice probability  $P_n(i, j)$  for property  $(i, j)$ , i.e.  $P_c$ , is

$$P_c := P_n(i, j) = \frac{e^{V_i' \mu^I}}{\sum_{i' \in I_n} e^{V_{i'}' \mu^I}} \cdot \frac{e^{\beta' x_{nij}}}{\sum_{j' \in J_{ni}} e^{\beta' x_{nij'}}}$$

where,

$$\begin{aligned} V_{i'}' &= \ln \sum_{j \in J_{ni}} e^{\beta' \mathbf{x}_{nij}}, \\ \mathbf{x}_{nij} &= (x_{nijp}, x_{njs1}, x_{njs2}, x_{njs3}, x_{njs4}, x_{njs5})' \end{aligned}$$

For simplicity, we now define

$$E = e^{\beta' \mathbf{x}_{nij}}; C = \sum_{j' \neq j; j' \in J_{ni}} e^{\beta' \mathbf{x}_{nij'}}; H = e^{V_i' \mu^l}; D = \sum_{i' \neq i; i' \in I_n} e^{V_{i'}' \mu^l}$$

Hence,

$$P_c = \frac{E}{E + C} \cdot \frac{H}{H + D}$$

Take the derivative of the terms defined above with respect to the chosen property's price variable  $x_{nijp}$  and get,

$$\frac{\partial E}{\partial x_{nijp}} = E\beta_p; \quad \frac{\partial C}{\partial x_{nijp}} = 0; \quad \frac{\partial H}{\partial x_{nijp}} = \frac{HE\mu^l}{E + C}\beta_p; \quad \frac{\partial D}{\partial x_{nijp}} = 0$$

Following simplification, we further obtain the derivative of  $P_c$  with respect to  $x_{nijp}$  is

$$\frac{\partial P_c}{\partial x_{nijp}} = \left[ \frac{HE}{(E + C)^2(H + D)} \left( C + \frac{DE\mu^l}{H + D} \right) \right] \beta_p$$

As shown in Table 3.3,  $\beta_p$  is negative and the terms in the brackets are all positive. Negative first derivative with respect to price implies that the choice probability decreases with prices. Therefore, we conclude the assumption in Theorem 1 that the property's display probability and choice probability are decreasing in price  $p(m, r)$  is satisfied based on our data set.

We use Theorem 1 in the numerical implementation of (3.10) as it reduces the set of prices that need to be searched over in the determination of  $p^*$ . We illustrate a numerical example in subsequent sections. In the following section we restrict daily prices to be fixed versus variable.

### 3.4.2 Daily Fixed Pricing

Similar to dynamic pricing we use (3.8) to define the probability that a randomly selected customer will purchase a room,  $P_s := P_s(p(t, r)) = P_d(p(t, r))P_b(t)P_c(p(t, r))$ . We use  $t$  to represent days before arrival, with  $m$  used to define periods in the dynamic pricing formulation. If we assume  $N$  customers arrive over day  $t$ , then the number of people who will make a purchase at a firm with posted price  $p(t, r)$  follows a binomial probability distribution. We denote this number by a random variable  $D_n(p(t, r))$  and, for  $0 \leq x \leq n$ ,

$$P(D_n(p(t, r)) = x) = \binom{n}{x} P_s(p(t, r))^x (1 - P_s(p(t, r)))^{n-x} \quad (3.23)$$

As the number of arrivals  $n$  is unknown with probability  $P(N = n)$ , the probability,  $f_{p(t, r)}$ , of the number of people willing to buy at the price  $p(t, r)$  equal to  $x$  becomes

$$f_{p(t, r)}(x) = \sum_{n=0}^{\infty} P(D_n(p(t, r)) = x) P(N = n). \quad (3.24)$$

As with dynamic pricing we assume customer arrivals follow a Poisson pro-

cess then,

$$\begin{aligned}
f_{p(t,r)}(x) &= \sum_{n=0}^{\infty} P(D_n(p(t,r))) = x) P(N = n) \\
&= \sum_{n=0}^{\infty} \binom{n}{x} P_s^x (1 - P_s)^{n-x} \frac{\lambda_t^n e^{-\lambda_t}}{n!} \\
&= \sum_{n=x}^{\infty} \binom{n}{x} P_s^x (1 - P_s)^{n-x} \frac{\lambda_t^n e^{-\lambda_t}}{n!} \\
&= \sum_{k=0}^{\infty} \frac{(k+x)!}{k!x!} P_s^x (1 - P_s)^k \frac{\lambda_t^{(k+x)} e^{-\lambda_t}}{(k+x)!} \\
&= \frac{\lambda_t^x e^{-\lambda_t}}{x!} P_s^x \sum_{k=0}^{\infty} (1 - P_s)^k \frac{\lambda_t^k}{k!} \\
&= \frac{\lambda_t^x e^{-\lambda_t}}{x!} P_s^x e^{\lambda_t(1-P_s)} \\
&= \frac{(\lambda_t P_s)^x e^{-\lambda_t P_s}}{x!}
\end{aligned} \tag{3.25}$$

With daily fixed prices a property displayed releases  $r$  rooms over the next  $T$  days,  $t = 0, 1, 2, \dots, T$ . The firm changes its daily price with the remaining capacity in order to maximize the total expected revenue. Define  $V^*(t, r)$  as the maximum achievable expected revenue with  $r$  rooms and  $t$  days remaining till the arrival day. Then  $V^*(t, r)$  is characterized by the following recursion,

$$V^*(t, r) = \max_{p(t,r)} \left\{ \sum_{x=0}^{\infty} [p(t, r) \min(r, x) + V^*(t-1, (r-x)^+)] f_{p(t,r)}(x) \right\}. \tag{3.26}$$

The boundary conditions are  $V^*(0, \cdot) = 0$ ,  $V^*(\cdot, 0) = 0$ .

We can define the expected marginal value of a room at day  $t$  as the following

$$\delta_x(t, r) = \frac{1}{x} [(V^*(t, r) - V^*(t, r-x))]$$

On each day, the property posts a price  $p(t, r)$  and sells  $x$ , up to  $r$  rooms. The following day the hotel updates its price, selling up to  $r - x$  rooms, until such a

time as it has no rooms left to sell for that arrival day,  $V^*(\cdot, 0) = 0$ , or the day has passed and the rooms are valueless,  $V^*(0, \cdot) = 0$ .

Unlike dynamic pricing we can not characterize all the properties of daily fixed pricing, but the following theorem greatly reduces the search space for finding optimal prices. The theorem is similar in nature to Theorem 3 from Lee and Hersh (1993). Lee and Hersh (1993) develop optimal booking limits in the presence of batch or group bookings. Lee and Hersh do not use price as a decision variable, but the sale of multiple items over a single decision period (batch bookings) is analogous to our daily fixed pricing policy.

**Theorem 2.** For fixed  $x$  and  $r$ ,  $\delta_x(t, r)$  increases in  $t$ .

*Proof.* We first show it is true for the case that  $x = 1$ , i.e.  $\delta_1(t, r) \geq \delta_1(t - 1, r)$ .

$$\begin{aligned} V^*(t, r) - V^*(t - 1, r) &= [V^*(t, r - 1) + v^t] - [V^*(t - 1, r - 1) + v^{t-1}] \\ &= [V^*(t, r - 1) - V^*(t - 1, r - 1)] + [v^t - v^{t-1}] \quad (3.27) \end{aligned}$$

where  $v^t$  denotes the expected revenue that can be generated by selling one room over the remaining  $t$  days. It is obvious that the expected revenue that a room can generate in  $t$  days is no less than the expected revenue that a room can generate in  $t - 1$  days. That means  $v^t - v^{t-1} \geq 0$ . Therefore, it follows from (3.27) that

$$V^*(t, r) - V^*(t - 1, r) \geq V^*(t, r - 1) - V^*(t - 1, r - 1)$$

then,

$$V^*(t, r) - V^*(t, r - 1) \geq V^*(t - 1, r) - V^*(t - 1, r - 1)$$

resulting in

$$\delta_1(t, r) \geq \delta_1(t - 1, r).$$

Then, when  $x > 1$ , we can write

$$\begin{aligned}
V^*(t, r) - V^*(t, r - x) &= [V^*(t, r) - V^*(t, r - 1)] \\
&\quad + [V^*(t, r - 1) - V^*(t, r - 2)] \\
&\quad + \dots + [V^*(t, r - x + 1) - V^*(t, r - x)] \quad (3.28)
\end{aligned}$$

From (3.27) above we note that

$$\begin{aligned}
\delta_1(t, r) \geq \delta_1(t - 1, r) &\implies \\
V^*(t, r) - V^*(t - 1, r) \geq V^*(t, r - 1) - V^*(t - 1, r - 1) &\implies \\
V^*(t, r) - V^*(t, r - 1) \geq V^*(t - 1, r) - V^*(t - 1, r - 1) & \\
&\text{for any } r > 1 \quad (3.29)
\end{aligned}$$

Thus, combining (3.28) and (3.29) gives

$$V^*(t, r) - V^*(t, r - x) \geq V^*(t - 1, r) - V^*(t - 1, r - x),$$

i.e.  $\delta_x(t, r) \geq \delta_x(t - 1, r)$  for any  $t$  given the fixed value of  $x$  and  $r$ .

Note that given the positive relationship between the optimal price and the marginal value, we can characterize that the optimal pricing policy is also increasing in the remaining selling time.

### 3.5. Numerical example

We next present a numerical example to illustrate the application of our choice based dynamic programming model under both daily fixed pricing and dynamic pricing as well as the revenue gains from moving to dynamic pricing

policies. In this example, we consider a 3 star hotel in Chantilly DC planning to sell 5 rooms over the last 3 days prior to a Monday arrival.

Table 3.8 summarizes the average number of requests received by day before arrival, the book-look ratios and the number of subperiods for  $\varepsilon = 0.05$  (probability of more than 1 request in a subperiod  $\leq 0.05$ ).

Table 3.8: Model Parameters - Monday Arrival 3 Star Hotel in Washington DC

	Days Before Arrival			
	0	1	2	3
Book-to-look ratio	0.041	0.039	0.040	0.037
Average requests	341	385	233	197
# of subperiods	958	1089	656	554

We use the parameters from Table 3.3 to build the nested logit model for estimating the probability of sale  $P_c$  given display with parameters from Table 3.7 used for the logistic regression to estimate the probability of being displayed  $P_d$ . The book-to-look ratios in Table 3.8 provide estimates of  $P_b$ . For the dynamic pricing model of section 3.4.1 the average requests (from Table 3.8) divided by the number of subperiods provides the arrival rate for arrivals. The average requests provides the rate for the fixed daily prices of section 3.4.2.

Table 3.9 displays optimal daily fixed prices. For example the firm would post a price of \$56 if it had 5 rooms to sell 3 days prior to arrival. If 2 reservations were accepted on the third day, they would increase prices to \$62 with 2 days left (and 3 rooms remaining). Consistent with the properties of dynamic pricing, Table 3.9 shows that prices are decreasing with increasing capacity and with decreasing time.

Table 3.9: Optimal Daily Fixed Prices

Rooms	Days Before Arrival			
	0	1	2	3
1	55	72	78	82
2	49	64	69	72
3	44	58	62	65
4	41	53	57	60
5	38	49	53	56

Figure 3.7 displays the same fixed prices as in Table 3.9 for the firm setting prices on the arrival date for 1 to 5 rooms with series labels F-1, F-2, F-3, F-4 and F-5 (F for fixed - number of rooms). The figure also displays optimal variable prices (in dollar price steps) throughout the arrival day, DBA 0 (period 958 the start of the day, 0 the end). If we look at the series for 1 room (F-1 for fixed and D-1 for dynamic) we see the dynamic price starts higher than the fixed decreasing gradually below the fixed price. For situations with more than 1 room the prices behave differently, with the dynamic price always less than the fixed.

Figure 3.8 shows the fixed price for the arrival day with 5 rooms to sell (F-5). The figure also displays five separate series for the dynamic pricing. Series D-5 assumes dynamic pricing at the start of the day with 5 rooms with D-4 assuming a room is reserved at the start of subperiod 750. The remaining series assume another reservation is accepted every 200 subperiods. Unlike daily fixed pricing, dynamic pricing sets prices knowing the firm will react to future sales, potentially raising prices as capacity is decreased.

Dynamic pricing offers increased revenue opportunities. As an estimate of



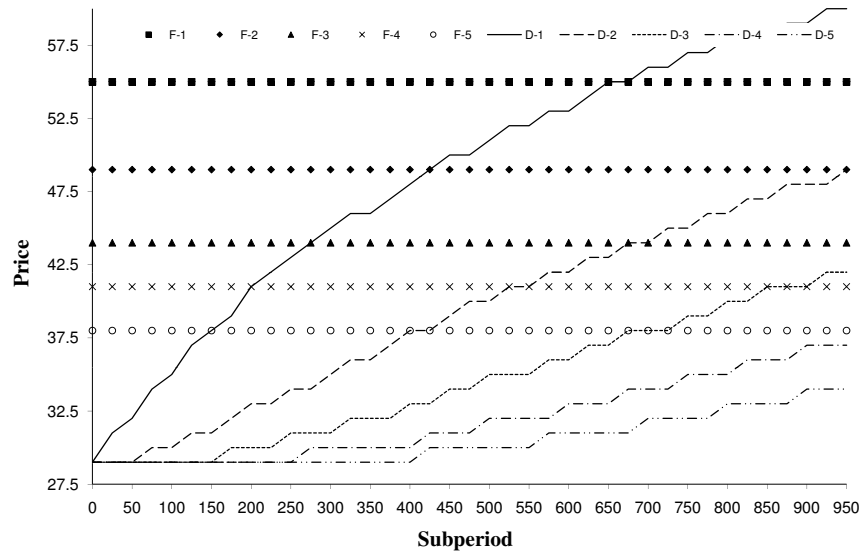


Figure 3.7: Comparing daily fixed prices versus dynamic prices for DBA=0 and 5 rooms

these revenue gains we simply evaluate (3.10) using optimal fixed daily prices versus optimal dynamic prices resulting in revenue gains of 5.0% from dynamic pricing.

### 3.6. Summary

Incorporating customer choice behavior into revenue management has been an active research area over the past several years. In this paper, we estimate a nested logit model on a data from firms selling hotel rooms at Hotwire.com to

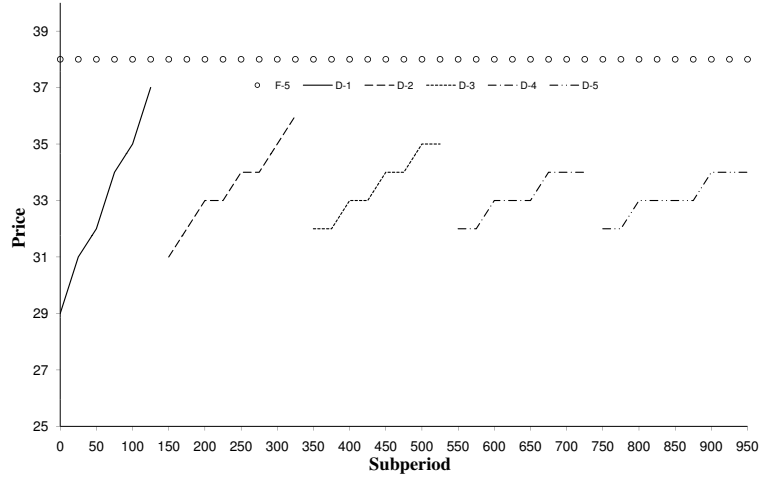


Figure 3.8: Daily fixed prices versus dynamic prices for DBA=0 and 5 rooms)

understand the customers' choice preferences on hotels located in Washington D.C. Our paper is unique in that it is the first choice based revenue management model that uses market level demand data (versus a single firm's sales data) to estimate the underlying demand models.

We model demand as a function of a property's prices using the result from the nested logit model and develop a choice-based dynamic programming model with pricing as the decision variable. We propose two stochastic dynamic programming formulations, one with daily fixed prices and one with fully dynamic prices. We provide a complete characterization of optimal dynamic prices and a partial characterization for optimal daily fixed prices. We show numerically that optimal fixed prices are consistent with the properties of

dynamic prices. We then use these two characterizations to estimate revenue impacts from daily fixed versus fully dynamic pricing policies.

Currently our formulation considers the firm selling capacity at the opaque reseller, ignoring the fact that they may simultaneously release inventory across numerous channels - both opaque like Hotwire (e.g. Priceline's Name-Your-Own-Price) and fully transparent (e.g. Expedia or Marriott.com). While our formulation can be extended to incorporate the opportunity cost of releasing rooms on other channels by including dual prices from standard revenue management models as in Anderson (2009), it currently does not directly include consumer channel choice behavior. An interesting direction for future research is to consider consumer choice behavior both across service providers as well as channel selection.

CHAPTER 4  
PRICING AND MARKET SEGMENTATION USING OPAQUE SELLING  
MECHANISMS

### 4.1. Online Travel Sales

The pricing of services (rooms, rental cars, airline seats, etc...) online has dramatically changed how service firms reach customers, with online travel sales now exceeding offline (or traditional sales channels). Initial thoughts about pricing online were very positive as firms had new channels to reach customers enabling increased opportunities for segmentation. Over time service providers have increased efforts to move customers back to company direct distribution channels (company websites and call centers) in an effort to control sales costs and commissions while maintaining direct contact with the customer to facilitate loyalty programs and other marketing efforts.

Hotwire and Priceline, unlike other online travel sales channels such as Expedia, Travelocity and Orbitz, offer customers opaque products with aspects of the service provider concealed until the transaction has been completed. Figure 4.1 shows a typical service provider listing (here hotels) on a full information channel like Expedia. Figures 4.2 and 4.3 display information available to someone using Hotwire's opaque mechanism. For instance a customer purchasing a hotel room through Hotwire can not specify the hotel they wish to stay at, but rather only its star rating and general location within the destination city. Customers do not know the identity or exact location of their non-refundable choice property until after purchase. Opaque travel sites offer service providers a convenient channel to segment customers and distribute discounted products

without cannibalizing or diluting full priced products. The opaque channels naturally segment customers as regular full price paying customers desiring to stay at the hotel of their choice with full cancellation flexibility are unique from those willing to purchase the discounted, non-refundable opaque product at the unknown service provider. Similar to the opaque posted price model of Hotwire, Priceline offers opaque services but without posted prices. Priceline's name-your-own-price model is similar to Hotwire where consumers, as shown in Figure 4.4, only know the star level and region for a hotel. On Priceline, consumers post bids for the opaque service as shown in Figure 4.5, having to then wait for the service provider to accept to reject their offer. For a more detailed description of Priceline's name-your-own-price model see Anderson (2009). While the illustrations provided in Figures 4.1-4.5 use hotels as examples, opaque services are also offered for other travel services. With air travel, the consumer is unaware of the itinerary (connections and layover durations) or airline and with rental cars, the consumer does not know the type of car or rental firm until after paying for the service. Lastminute.com, another online travel agent, also offers opaque posted price services similar to those of Hotwire.

The level of opacity varies across the different opaque channels as some choose to offer cancellation opportunities as in the case of Lastminute.com, provide user generated feedback as in the case of Hotwire.com, or list some of the amenities offered by the service provider. Similarly the degree of opacity may also be impacted by the market, as markets with fewer similar competitors offer decreased opacity over markets with a larger number of service providers.

Opaque selling has recently started to receive interest in the the academic literature, most of the early research has focused on models similar to Priceline's

The Dupont Hotel ★★★★★  
 Washington, DC Area: Dupont Circle  
 Overlooking Dupont Circle in one of Washington's most desirable neighborhoods, this hotel is about 1 block...  
 Photos | Features & Amenities | Location

Traveler Opinion: 4.2 out of 5 (75+ reviews)

Expedia Special Rate: \$149 avg per night

Special Offer: Last Minute Deal Save an additional 25%.

Room type	Fri	Avg per night
Superior room	<del>\$499</del> \$149	\$149.25 <a href="#">SELECT</a>
Superior 2 Double-Standard	<del>\$499</del> \$149	\$149.25 <a href="#">SELECT</a>

[Hide room pricing](#) | Book online or call 1-800-551-2409

Figure 4.1: Typical Full Information Hotel Listing

★★★★½ \$165 per room per night <a href="#">Continue</a>	4.5-star hotel in Dupont Circle - Embassy Row Area map and details ▼	Hotwire Hot Rate <input checked="" type="checkbox"/> Lowest Price Guaranteed <input checked="" type="checkbox"/> Best value
★★★★ \$129 per room per night <a href="#">Continue</a>	4-star hotel in Dupont Circle - Embassy Row Area map and details ▼	Hotwire Hot Rate <input checked="" type="checkbox"/> Lowest Price Guaranteed
★★★ \$108 per room per night <a href="#">Continue</a>	3-star hotel in Dupont Circle - Embassy Row Area map and details ▼	Hotwire Hot Rate <input checked="" type="checkbox"/> Lowest Price Guaranteed
★★ \$99 per room per night <a href="#">Continue</a>	2-star hotel in Dupont Circle - Embassy Row Area map and details ▼	Hotwire Hot Rate <input checked="" type="checkbox"/> Lowest Price Guaranteed

Figure 4.2: Posted Opaque Hotel Listing

name-your-own-price (NYOP) bidding mechanism where customers post bids for opaque services. Anderson (2009) provides a detailed background on the nature of Priceline's NYOP model as well as a dynamic programming based model for the setting of prices by firms on Priceline. Fay (2004) develops a stylized model of a monopolist firm using a NYOP channel and investigates whether repeat bidding should be allowed. Strictly speaking, Priceline does not allow repeat bidding within a 24 hour period but there are numerous methods to cir-

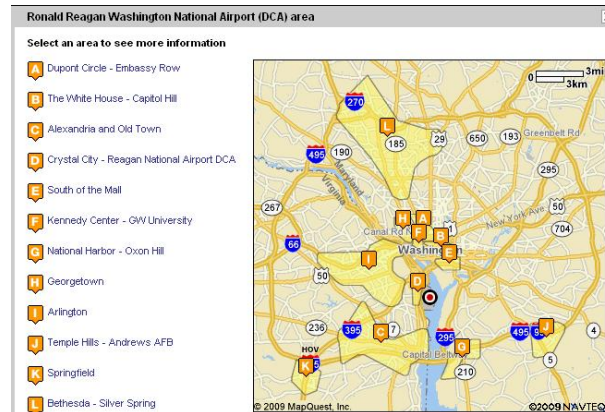


Figure 4.3: Posted Opaque City Areas

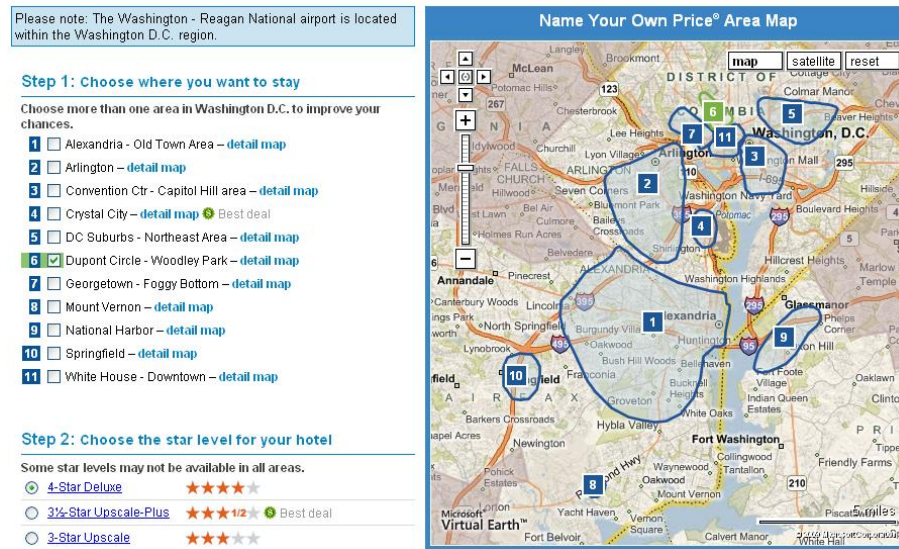


Figure 4.4: Opaque Bidding Hotel Listing

cumvent this limitation, see BiddingforTravel.com for examples. Fay indicates that partial repeat bidding, i.e. repeat bidding by knowledgeable customers may be less profitable than complete repeat bidding. Fay (2008) extends the monopolist model to a duopoly model with firms pricing into two consumer segments. One segment is loyal to a particular service provider, the second has preferences distributed between the two firms along a line as in the traditional Hotelling

**Step 3: Name Your Own Price® (per room night)**

Total charges, including **taxes and service fees**, are shown on the next page.

<b>Name Your Own Price®</b> Per Room, Per Night (\$USD) \$ <input type="text"/> .00	Median <b>retail price</b> for a 4 star hotel in the area selected is <b>\$199</b> . Name a lower price, or <a href="#">click here</a> to shop and compare prices.
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Figure 4.5: Submitting Opaque Bid

model (Hotelling, 1929). Fay (2008) is the first paper to investigate how product opacity affects the market. Fay studies two competing service providers selling products to two types of customers (business and leisure) on both an opaque posted price channel and a traditional distribution channel. Fay shows that opaque selling benefits the monopoly service provider when customers have heterogenous values for products. Shapiro and Shi (2008) extend the model of Fay (2008) to  $N$  firms with the number of firms indicating the degree of opacity - uncertainty in knowledge of service provider increases with number of firms. Shapiro and Shi focus on providing a rationale for opaque selling. They explain why service providers are willing to distribute products through opaque travel sites such as Priceline and Hotwire and lose the advantage of product differentiation.

Hann and Terwiesch (2003) use data from a European NYOP retailer to investigate consumer transactions costs (the cost of resubmitting bids) of using a repeat bidding NYOP channel. In a related paper Spann et al. (2004) investigate consumers' frictional or transactions costs as well as their willingness to pay using data from a German NYOP seller of flights from Germany to Spain. Wang et al.(2009) develop a game theoretic model of a supplier using both regular posted price full information channels as well as a NYOP channel to reach



heterogeneous customers. They develop a two-stage game where suppliers set posted prices in period 1 and after observing demand in period 1, set minimally acceptable prices at the NYOP channel in period 2. Posted prices are rigid in period 2. Consumers observe posted prices in the first period then decide to buy or bid in period 2. The rigidity of posted prices combined with demand uncertainty results in the NYOP channel generating improved revenues for the service provider. Wilson and Zhang (2008) look at a retailer setting prices on a NYOP channel. They develop  $\epsilon$  optimal policies for the retailer that encourage the customer to bid their maximum reservation price.

Related research looks more generally at opaque selling where prices are posted but some aspect of the service or service provider is hidden i.e. the selling mechanism similar to that provided by Hotwire.com. Jiang (2007) develops a Hotelling type model to illustrate how a firm should price on regular full information channels versus opaque channels. Jiang indicates that opaque selling can be Pareto improving for both customers and suppliers when customers are differentiated in their willingness to pay. Jiang compares opaque selling and regular selling (selling full-information products), providing insight when to implement opaque selling. Jerath, Netessine and Veeraraghavan (2007) compare opaque selling with last-minute direct selling and obtain the conditions under which opaque selling is preferred. In their model two firms of equal capacity offer a differentiated service via three channels: regular posted price, posted last-minute sales, and last-minute sales through an opaque intermediary. Their goal is to investigate under what market conditions a firm should directly offer last-minute discounts versus offer those discounts through an intermediary. Jerath et al. relax the posted price rigidity of Wang et al. (2005) through introduction of the direct last-minute discounts. They conclude that direct last-minute selling is

preferred over the opaque intermediary when consumer valuations are high or if the service offerings are relatively homogeneous.

While there is an extensive body of research on the use of auctions, very little of this research looks at the simultaneous use of auctions and posted prices. Firms can use auctions to reach customers whom may not otherwise purchase, as posted prices may be too high. Conversely auctions potentially dilute revenues as customers willing to pay posted (full prices) may purchase (at lower prices) via the auction. The opaque nature of Priceline's NYOP model helps to avoid this dilution. Etizon, Pinker and Seidmann (2006) is one of the few auction related papers that looks at the simultaneous use of auctions and posted prices. Similar to our development they look at a firm with excess supply facing consumers who strategically choose to purchase at posted prices or bid (resorting to posted prices if their bid fails). Different from our model, consumers do not face any product opacity with the auction but do incur a waiting cost associated with bidding. Van Ryzin and Vulcano (2004) look at firm using posted prices as well as an auction mechanism, unlike our model of endogenous channel choice (strategic customers similar to Etizon et al.) they assume separate streams of customers to each channel with the seller deciding on inventory allocation across the channels. Huang and Sasic (2011) and Caldentey and Vulcano (2007) also look at firms using auctions in concert with posted prices. Both assume customers arrive according to a poisson process and focus on dynamic inventory management strategies for the seller. Huh and Janakiraman (2008) illustrate the optimality of  $(s,S)$  inventory management policies for firms using several different selling mechanisms (including name-your-price mechanisms) in settings where firms can replenish inventory at prescribed costs. Cai et al. (2009) investigate the potential benefits of a NYOP retailer in addition to a

posted price channel with consumers allowed to return to posted price channels upon failed bid attempts.

We develop a stylized model of consumers looking to acquire travel services through either full information or opaque channels (both posted price and bidding). Consumers choose their channel or sequence of channels (in the case of bidding first followed by posted prices) that maximizes their surplus. Our paper is unique from the literature in that it is the only paper that investigates a firm using two opaque (posted and bidding) channels simultaneously with regular full information posted price channels. Second, prior research assumes two or more exogenous customer segments (i.e. business and leisure) with the opaque channels targeted at the leisure or price sensitive segment; whereas we develop endogenous consumer segments where consumers choose the channel of their choice by maximizing their surplus. Our goal is to illustrate how opaque channels naturally segment consumers as well as how firms should use and price into these channels as a function of the degree of their opacity. We also discuss the segmentation and policy changes induced by capacity constraints. We show that simultaneously selling through regular and opaque channels even in the presence of tight capacity constraints helps firms to segment consumers, differentially pricing into different willingness to pay segments and improve revenues (over the absence of opaque pricing).

## **4.2. Model Development**

We develop a model of a firm selling to strategic consumers - consumers are strategic as they choose the channel or sequence of channels which maximizes

their surplus. The seller can potentially offer its products across three selling mechanisms: a posted full information market, posted opaque market with certain aspects of the product hidden and a name your price opaque auction mechanism. Unlike previous research which assumes exogenous consumer behavior we model endogenous consumer behavior where all consumers act strategically as they optimally choose the channel (or sequence of channels) that maximizes their surplus. For ease of exposition we will refer to the full information channel as the regular (REG), the opaque posted price channel as opaque (OPQ) and the opaque channel with bidding as BID. For comparison purposes, think of our regular channel as a firm's website (Marriott.com, Hilton.com or USAirways.com) or a typical online travel agent similar to Expedia, Orbitz or Travelocity, the posted opaque channel analogous to Hotwire.com, and our bidding model similar to Priceline's name-your-own-price model. We do not model competition in the full information market as the firm is selling a differentiated/branded product desired by consumers.

Each customer  $i$  looking to acquire service has an independent reference price or valuation  $v_i$  for the service provider. Similar to Wang et al. (2009) we assume  $v_i$  uniformly distributed between 0 and 1, i.e. its density function  $f(v_i)$  is 1 for  $0 \leq v_i \leq 1$  and 0 otherwise. The service provider posts a price  $P_1$  on the regular channel and fully discloses all service provider characteristics. The service provider posts price  $P_2$  on the opaque posted price channel and reveals the full information until after the purchase. The service provider also sets a threshold price  $R$  on the opaque bidding channel. The customer, if they choose to bid, bids  $B_i$  on the bidding channel. Similar to Hann and Terwiesch (2003), Spann et al. (2004) and Ding et al. (2005), with limited knowledge of the value of the threshold  $R$ , customers expect  $R$  to be distributed uniformly over  $[0, 1]$ . As

a result customers believe their bid of  $B$  will be accepted by the service provider with a probability of  $B$ .

When a consumer pays  $P_1$  at the regular full information channel they are purchasing the product from the service provider of choice, here assuming the consumer has an affinity for this branded service provider. When the consumer pays  $P_2$  at a posted opaque channel they know they are receiving a similar product but they don't know from which service provider - e.g. could be any of 10 3-Star hotels in Times Square NYC. Typically posted price opaque channels like Hotwire.com display online the service provider whom has provided them the lowest price - e.g. if all 10 of the aforementioned 3-Star Times Square hotels offered inventory to Hotwire only the one with the cheapest price would be posted with the opportunity for a sale. Which property is displayed would change over time as transactions occur and inventory is sold. Priceline's opaque bidding channel behaves in a similar fashion except the consumer submits an offer,  $B_i$ , for a 3-Star Times Square hotel, Priceline then randomly selects from the firms that have provided it with inventory to see if they have a price that is less than the consumer's offer price. Priceline randomly rotates through all the qualifying hotels (3-Star Times Square) until either a hotel with a price low enough is found or no service provider meets the consumer's bid. Online boards such as BiddingForTravel.com provide resources and historic bid results to help consumers in determining how to bid on Priceline. For a more exhaustive discussion of Priceline see Anderson (2009).

The service provider looks to augment its full information channel with the opaque channels in an effort to sell surplus inventory. The service provider looks to use the opaque channels even though they yield considerably lower

revenues (typical discounts at Hotwire and Priceline range from 25-50%). Figure 4.6 shows a set of sample reservations buildup for a 3.5 star hotel in Dupont Circle Washington DC. The figure shows the average percentage of reservations over the last week prior to arrival for 6 weeks of arrival dates in the fall of 2008. The figure displays total reservations as well as those through each of Hotwire and Priceline. As can be seen from the figure Hotwire and Priceline are typically only used very close to the arrival day. Virtually no reservations are accepted on opaque channels prior to 7 days before arrival whereas approximately half of total reservations have been received prior to the last week. The service provider is using the deeply discounted opaque channels to sell distressed inventory, inventory that would otherwise not be sold, over these final few days prior to arrival. During these last few days prior to the service becoming worthless (hotel bed not occupied or airline seat flying empty) the firm is in essence pricing without capacity considerations (able to meet all demand). Whereas earlier on in the selling process (several weeks or months prior to arrival at the hotel or departure of the aircraft) the firm may not use opaque channels as it prices in consideration of capacity constraints - hoping to sell all inventory at higher prices to the brand loyal customers on the full information channels. As we will also see in later sections, the firm also tends not to use the opaque channels if they are not very opaque. The opaque channels become increasingly less opaque earlier on in the selling process as fewer firms may tend to use them - with opacity as in Shapiro and Shi (2008) directly related to the number of service providers using the opaque channels.

In the following sections we outline optimal prices and the resulting market segmentation for a service provider who has the opportunity to release their products on the regular full information channel, an opaque posted price chan-

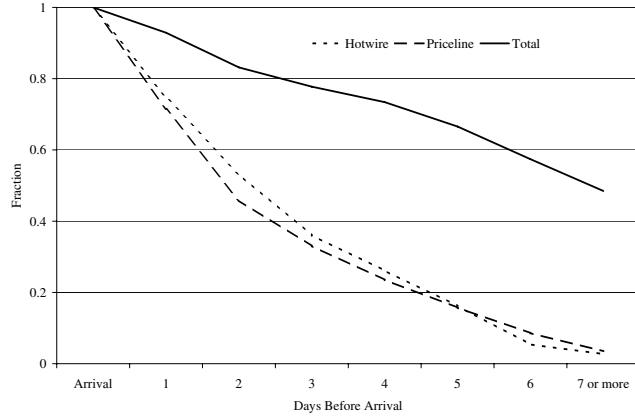


Figure 4.6: Reservations buildup at Hotwire, Priceline and all channels for 3.5 star DC hotel

nel and an opaque channel with bidding. We illustrate our modeling approach when the service provider chooses to list only on the full information channel, optimal prices and the resulting revenue provide a basis to later compare multi-channel strategies. Initially we focus on a firm with no capacity constraint, later extending the formulation to a firm where demand exceeds capacity. For ease of presentation, and without loss of generality, all revenues are normalized to a market of one.

#### 4.2.1 Customer Segmentation

The service provider chooses to release products only on the REG and set its price as  $P_1$ . Consumer  $i$  has surplus  $CS_i = v_i - P_1$ , so only consumers with valuation higher than the price  $P_1$  will purchase on this channel (Table 4.1 sum-

marizes all model notation).

Table 4.1: Notations in the model

$v_i$	consumer $i$ 's valuation of the product
$P_1$	price set by the service provider on the REG channel
$P_2$	price set by the service provider on the OPQ channel
$R$	bidding threshold set by the service provider on the BID channel
$d_1$	the discount factor for purchasing on OPQ
$d_2$	the discount factor for bidding on BID
$B_{i1}(v_i)$	the bid of consumer $i$ in the segment of BID then REG market segment
$B_{i2}(v_i)$	the bid of consumer $i$ in the segment of BID then OPQ market segment
$B_{i3}(v_i)$	the bid of consumer $i$ in the segment of BID only market segment
$V_1, V_2, V_4$	the critical value points in the market segmentation
$V_3, V_5, V_6$	the critical value points in the revenue segmentation
$C$	the capacity constraint

Therefore, the expected revenue for the service provider  $\pi$  is given by

$$\pi = \int_{P_1}^1 P_1 f(v_i) dv_i = P_1(1 - P_1) = P_1 - P_1^2 \quad (4.1)$$

Taking the derivative of  $\pi$  with respect to  $P_1$  and setting it to be zero, we can solve for the optimal price should be posted on REG:  $P_1^* = \frac{1}{2}$ .

Since  $\frac{d^2\pi}{dP_1^2} = -2 < 0$ , we substitute  $P_1^*$  back into (4.1) and get the maximum revenue  $\pi^* = \frac{1}{4}$ . Moreover, from (4.1), it is straightforward to see that the maximum revenue is concave in the prices. Figure 4.7 summarizes the segmentation created by only pricing on the REG.

The service provider now release products on the REG, OPQ and BID simultaneously. They set price  $P_1$  on REG,  $P_2$  on OPQ, a bidding threshold i.e.



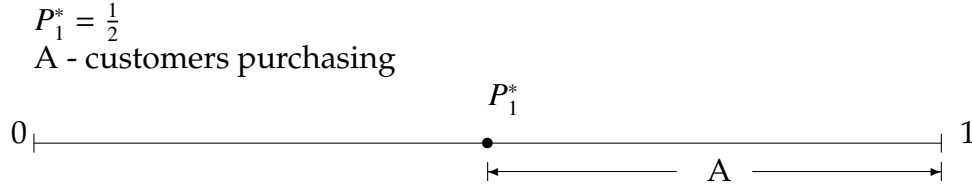


Figure 4.7: Segmentation resulting from full information posted prices

the minimum acceptable bid  $R$  on BID, which is unknown to the consumers. However consumers expect  $R$  to follow a uniform distribution over  $[0, 1]$ .

The consumer surplus from purchasing on the REG is  $CS_i = v_i - P_1$ . To allow comparison of consumer surplus across channels we adopt a utility framework, where the utility,  $U(CS)$ , resulting from a surplus  $CS$  is assumed to be linear, i.e.  $U(CS) = d_j CS + b_j$  for  $j = 0, 1, 2$  with  $j$  being a channel specific index ( $0 = \text{REG}, 1 = \text{OPQ}, 2 = \text{BID}$ ). For simplicity, but without loss of generality, we assume  $b_j = 0$  and  $d_0 = 1$  for consumers acquiring service from the full information channel. The utility for a consumer purchasing on REG is simply  $U(CS_i) = v_i - P_1$ . As the consumer is not fully aware of all the service provider's characteristics when purchasing through OPQ we discount the consumer surplus from purchasing on OPQ. Let  $d_1$  denote the discount factor for purchasing on OPQ resulting in utility  $U(CS_i) = d_1(v_i - P_2)$  from purchasing on OPQ, where  $0 \leq d_1 < 1$ . Here  $1 - d_1$  represents the opacity of the opaque channel, implying as  $d_1$  approaches 1 the channel becomes less opaque as the consumer discounts the surplus less. Similarly, we denote the degree of opacity of the products on the BID channel by  $1 - d_2$ . As indicated in Shapiro and Shi (2008) that the degree of opacity is related to the numbers of competitors using the opaque channel. More specifically, for example, if there are  $N$  service providers listing their prod-

ucts on the opaque channel, i.e. not disclosing their identity, then in general the consumer's chance of purchasing from one of them is  $\frac{1}{N}$ . And so, the degree of opacity can be interpreted as a function of  $\frac{1}{N}$ .

If consumer  $i$ 's valuation  $v_i$  satisfying  $v_i - P_1 \geq d_1(v_i - P_2)$  and  $v_i \geq P_1$ , then the consumer will prefer to purchase on the REG versus OPQ. If  $v_i - P_1 < d_1(v_i - P_2)$  and  $v_i \geq P_2$ , then they will choose OPQ to make the purchase. The customer will be indifferent to purchasing on REG and OPQ when  $v_i = \frac{P_1 - d_1 P_2}{1 - d_1} := V_1$ .

Some consumers may bid first and switch to the REG channel if their bid gets rejected and their valuations are higher than  $P_1$ . Suppose  $B_i$  is the bid that consumer  $i$  submits to BID, and he expects it to be accepted with a probability of  $B_i$ . If the bid is rejected (with probability of  $1 - B_i$  in consumers' belief), the consumer will go to the REG and purchase the product at  $P_1$ . Given  $v_i \geq P_1$ , the utility for consumer  $i$  is then the sum of the utilities from a possible opaque bidding purchase and in the case their bid is rejected the utility from purchasing at regular prices,

$$U(CS_i) = d_2(v_i - B_i)B_i + (1 - B_i)(v_i - P_1). \quad (4.2)$$

As  $U(CS_i)$  is a concave quadratic function of  $B_i$ , it is straightforward to show  $U(CS_i)$  is maximized when  $B_i = B_{i1}^*(v_i)$ , where

$$B_{i1}^*(v_i) = \frac{P_1 - (1 - d_2)v_i}{2d_2}. \quad (4.3)$$

As bids must be positive, i.e.  $B_i > 0$ , this results in

$$B_{i1}^*(v_i) > 0 \implies v_i < \frac{P_1}{1 - d_2} := V_2, \quad (4.4)$$

It is easy to show that that the optimal bid is less than  $P_1$  and is decreasing in the

opacity degree on the BID channel as the products on the BID channel become less valuable for the customers while the BID channel becomes more opaque.

For the consumer  $i$  who chooses to bid  $B_{i1}^*(v_i)$ , by substituting the bid back into (4.2) we obtain their maximum expected surplus  $U(CS_{i1}^*(v_i)) = d_2(B_{i1}^*(v_i))^2 + (v_i - P_1)$ , which exceeds the utility,  $(v_i - P_1)$ , from buying directly from the posted full information channel. Therefore, consumers with valuation  $P_1 \leq v_i < V_2$  will choose to bid  $B_{i1}^*(v_i) = \frac{P_1 - (1-d_2)v_i}{2d_2}$  first and then go to the REG channel if their bids fails.

However, from the service provider's perspective, the bid  $B_{i1}^*(v_i)$  will be accepted only if  $B_{i1}^*(v_i) > R$  i.e.  $v_i < \frac{P_1 - 2d_2R}{1-d_2} := V_3$ . This means that customers with valuations  $v_i \in [V_3, V_2)$  will lose the bid (note that they do not know it before they bid) and go back to purchase on REG. Customers with valuation  $v_i \in [P_1, V_3)$  will win the bid.

A subset of consumers may choose to bid first and switch to purchase at the OPQ channel if their bid is rejected and  $v_i \geq P_2$ . Assume  $B_i$  is the bid that consumer  $i$  submits to BID and he believes the accepting probability is  $B_i$ . If the bid is rejected, the consumer will go to the OPQ and purchase at  $P_2$ . Given  $v_i \geq P_2$ , the surplus for consumer  $i$  is

$$U(CS_i) = d_2(v_i - B_i)B_i + (1 - B_i)(v_i - P_2) \quad (4.5)$$

It is straightforward to show  $U(CS_i)$  is maximized when  $B_i = B_{i2}^*(v_i)$ , where

$$B_{i2}^*(v_i) = \frac{d_1P_2 - (d_1 - d_2)v_i}{2d_2}. \quad (4.6)$$

As consumers bids must be positive, i.e.  $B_i > 0$ , this results in

$$B_{i2}^*(v_i) > 0 \implies v_i < \frac{d_1P_2}{d_1 - d_2} := V_4, \quad (4.7)$$

One can easily show that  $B_{i2}^*(v_i)$  is less than  $P_2$  and we now take the first derivative of  $B_{i2}^*(v_i)$  with respect to  $d_1$  and  $d_2$  respectively and get

$$\frac{dB_{i2}^*(v_i)}{d d_1} = -\frac{v_i - P_2}{2d_2} \leq 0 \text{ since } v_i \geq P_2, \quad (4.8)$$

$$\frac{dB_{i2}^*(v_i)}{d d_2} = \frac{d_1(v_i - P_2)}{2d_2^2} \geq 0 \text{ since } v_i \geq P_2. \quad (4.9)$$

The optimal bid for customers who choose bid first and go purchase at the OPQ channel if the bid fails is decreasing in the opacity degree on BID, but increasing in the opacity degree on OPQ. This is because the products on the BID channel becomes less valuable for the customers while the BID channel becomes more opaque, but becomes more valuable when the OPQ channel becomes more opaque.

We substitute  $B_{i2}^*(v_i)$  back and obtain the maximum expected utility for consumer  $i$  is  $U(CS_{i1}^*(v_i)) = d_2(B_{i2}^*(v_i))^2 + d_1(v_i - P_2)$ , which exceeds the surplus,  $d_1(v_i - P_2)$ , from buying directly from the OPQ channel as desired.

However, similar to the segment of BID then purchase at REG after the bid fails, the bid  $B_{i2}^*(v_i)$  will be accepted only if  $B_{i2}^*(v_i) > R$  i.e.  $v_i < \frac{d_1 P_2 - 2d_2 R}{d_1 - d_2} := V_5$ . Thus customers with valuations  $v_i \in [V_5, V_4)$  will lose the bid (again, they do not know it before they bid) and switch to purchase at OPQ. Customers with valuations  $v_i \in [P_2, V_5)$  will win their bid.

For consumers with valuations lower than  $P_2$ , their only choice is to bid. Their surplus is  $U(CS_i) = d_2(v_i - B_i)B_i$ , which is maximized with  $B_i = B_{i3}^*(v_i) = v_i/2$ . Service provider will only accept the bid when  $B_{i3}^*(v_i) > R$  i.e.  $v_i > 2R := V_6$ . This means that customers with valuations  $v_i \in [0, V_6)$  will lose the bid and leave empty handed, while customers with valuations  $v_i \in [V_6, P_2)$  will win their bid and get the product.

We now summarize the consumer self-selected market segmentation when the service provider can list products on all three channels : REG, OPQ, and BID and illustrate it by using critical points  $V_1, P_1, V_2, V_4, P_2$ . Based on the relationship between the discount factors  $d_1, d_2$  and prices  $P_1, P_2$  posted on channels REG and OPQ there are two cases of consumer market segmentation as follows:

**Case I.**  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq 0 \Leftrightarrow V_4 \geq V_2 \geq V_1$ .

The three channels REG, OPQ and BID partition consumers into four potential segments as shown in Table 4.2.

Table 4.2: Market Segmentation - Case I

Valuation ( $v_i$ )	Segment	Critical Points		
		$1 > V_2$	$V_2 \geq 1 > V_1$	$V_1 \geq 1$
$[V_2, 1]$	REG	Present	Absent	Absent
$[V_1, \min(V_2, 1))$	BID then REG	Present	Present	Absent
$[P_2, \min(V_1, 1))$	BID then OPQ	Present	Present	Present
$[0, P_2)$	BID	Present	Present	Present

Where, REG denotes buying on REG only; BID then REG denotes bidding then purchasing at REG if bid fails; BID then OPQ denotes bidding then purchasing at OPQ if bid fails; BID denotes bidding only.

**Case II.**  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0 \Leftrightarrow V_4 < V_2 < V_1$  and  $V_1 \geq P_1 \geq P_2$ .

Note that this case only exists when  $d_1 > d_2$ , since when  $d_1 \leq d_2$ ,  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq 0$ .

In this case, the three channels REG, OPQ and BID partition consumers into four potential segments as displayed in Table 4.3.

Table 4.3: Market Segmentation - Case II

Valuation ( $v_i$ )	Segment	Critical Points		
		$1 > V_1$	$V_1 \geq 1 > V_3$	$V_3 \geq 1$
$[V_1, 1]$	REG	Present	Absent	Absent
$[V_3, \min(V_1, 1))$	OPQ	Present	Present	Absent
$[P_2, \min(V_3, 1))$	BID then OPQ	Present	Present	Present
$[0, P_2)$	BID	Present	Present	Present

Where, REG denotes buying on REG only; OPQ denotes buying on OPQ only; BID then OPQ denotes bidding then purchasing at OPQ if bid fails; BID denotes bidding only. Figure 4.8 displays the market segmentation of these two cases.

#### 4.2.2 Optimal Service Provider Policies

In this section, we solve for the optimal prices and threshold set on the channels REG, OPQ and BID respectively and the resulting maximum expected revenue for a service provider under both segmentation cases discussed previously. As mentioned before we assume all revenues are normalized to a market of one, as such expected revenue values are per customer. In this section we allow the firm to optimally set  $P_1$ ,  $P_2$  and  $R$ , the setting of prices and thresholds then dictates the segmentation of consumers (and the resulting optimal revenue).

### Case I

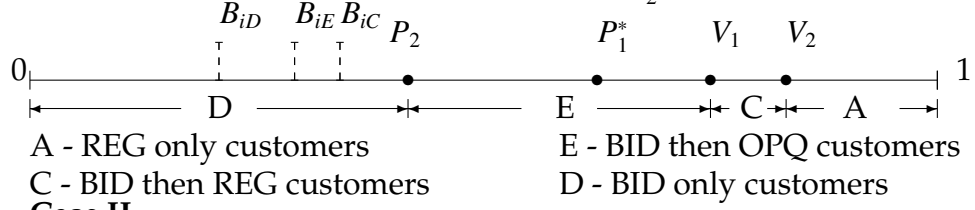
$$V_1 = \frac{P_1 - d_1 P_2}{1 - d_1}$$

$$V_2 = \frac{P_1}{1 - d_2}$$

$$B_{iC} = \frac{P_1 - (1 - d_2)v_i}{2d_2}$$

$$B_{iE} = \frac{d_1 P_2 - (d_1 - d_2)v_i}{2d_2}$$

$$B_{iD} = \frac{v_i}{2}$$



### Case II

$$V_1 = \frac{P_1 - d_1 P_2}{1 - d_1}$$

$$V_4 = \frac{d_1 P_2}{d_1 - d_2}$$

$$B_{iE} = \frac{d_1 P_2 - (d_1 - d_2)v_i}{2d_2}$$

$$B_{iD} = \frac{v_i}{2}$$

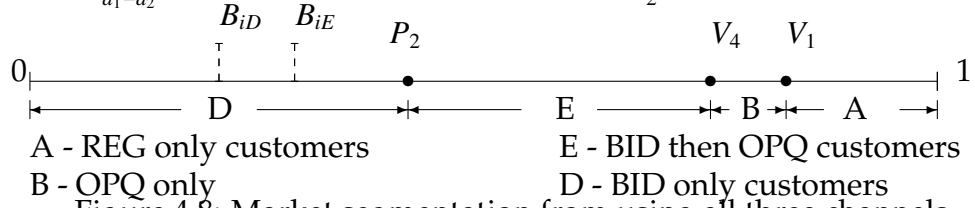


Figure 4.8: Market segmentation from using all three channels

As discussed earlier that  $B_{i1}^*(v_i)$ ,  $B_{i2}^*(v_i)$ , and  $B_{i3}^*(v_i)$  are the optimal bids for the consumers in the segments of BID and purchase at REG if the bid fails, BID and purchase at OPQ if the bid fails and BID only respectively. However, from the perspective of the service provider, those bids can be accepted only when they are more than the threshold  $R$ , i.e.  $B_{i1}^*(v_i) > R$ ,  $B_{i2}^*(v_i) > R$ , and  $B_{i3}^*(v_i) > R$ . This implies consumers in those three segments will win the bidding if their valuations  $v_i < V_3$ ,  $v_i < V_5$ , and  $v_i > V_6$  respectively. Hence,  $V_3$ ,  $V_5$ ,  $V_6$  are critical points for determining which channels the revenue is actually coming from. Recall that  $V_1, P_1, V_2, V_4, P_2$  are the critical points for consumer market segmentation and based on the relations among  $d_1, d_2, P_1$ , and  $P_2$  there are the two segmentation cases. From the perspective of the service provider, We now have several scenarios in each segmentation case as a function of  $d_1, d_2, P_1, P_2$  and  $R$ , and display the scenarios using critical points  $V_1, V_2, V_3, V_4, V_5, V_6, P_1$ , and  $P_2$  as discussed in

the following.

**Case I.**  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq 0 \Leftrightarrow V_4 \geq V_2 \geq V_1$ .

Recall that the consumer segmentation given in Table 4.2.

There are three revenue scenarios in this segmentation case.

*Case I - Scenario I.*  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq -2d_2R(1 - d_1)$

Consumers in the first segment  $[V_2, 1]$  (if  $V_2 < 1$ ) buy on REG directly. It is straightforward to check that  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq -2d_2R(1 - d_1)$  is equivalent to

$$\frac{P_1 - 2d_2R}{1 - d_2} \geq \frac{P_1 - d_1P_2}{1 - d_1}, \text{ i.e. } V_3 \geq V_1.$$

Thus the segment BID then REG ( $[V_1, \min(V_2, 1))$ ) is divided into two groups of customers with the first group  $v_i \in [\min(V_3, 1), \min(V_2, 1))$  purchases on REG and second group  $v_i \in [V_1, \min(V_3, 1))$  wins the bid.

One can also easily show that  $V_3 \geq V_1 \Rightarrow V_5 \geq V_1$ , then in the segment of BID then OPQ ( $v_i \in [P_2, \min(V_1, 1))$ ), all consumers will win their bids since their optimal bids are above the threshold  $R$  as long as their valuation  $v_i \leq V_5$ .

$V_5 \geq V_1 \geq P_1 \geq P_2$  (since if  $P_1 < P_2$ , no one would buy on REG, i.e. there is no REG only segment existing)  $\Rightarrow P_2 \geq V_6$ , then in the segment of bidding only, consumers with valuations  $v_i \in [V_6, P_2)$  win their bid and consumers with valuations  $v_i \in [0, V_6)$  lose. Table 4.4 summarizes this revenue scenarios.

Therefore, if  $1 \geq V_3 \geq V_1$  i.e.  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq -2d_2R(1 - d_1)$  and  $P_1 - 2d_2R \leq 1 - d_2$ , then the expected revenue  $\pi$  for the service provider in this



Table 4.4: Revenue scenario, Case I - Scenario I

Valuation ( $v_i$ )	Transaction channel	Critical Points		
		$1 > V_3$	$V_3 \geq 1 > V_1$	$V_1 \geq 1$
$[V_3, 1]$	REG ( $P_1$ )	Present	Absent	Absent
$[V_1, \min(V_3, 1))$	BID ( $B_{i1}^*(v_i)$ )	Present	Present	Absent
$[P_2, \min(V_1, 1))$	BID ( $B_{i2}^*(v_i)$ )	Present	Present	Present
$[V_6, P_2)$	BID ( $B_{i3}^*(v_i)$ )	Present	Present	Present

scenario is:

$$\begin{aligned}
 \pi &= \int_{V_3}^1 P_1 f(v_i) dv_i + \int_{V_1}^{V_3} B_{i1}^*(v_i) f(v_i) dv_i + \int_{P_2}^{V_1} B_{i2}^*(v_i) f(v_i) dv_i + \int_{V_6}^{P_2} B_{i3}^*(v_i) f(v_i) dv_i \\
 &= \frac{[-d_1(P_1 - P_2)^2 - 4(-1 + d_1)d_2^2 P_1(-1 + 2R) + d_2((-3 + 4d_1)P_1^2 + d_1P_2^2) - 2P_1(-2 + d_1(2 + P_2)) - 4R^2 + 4d_1R^2]}{4(-1 + d_1)(-1 + d_2)d_2}
 \end{aligned} \tag{4.10}$$

where, recall that

$$\begin{aligned}
 B_{i1}^*(v_i) &= \frac{P_1 - (1 - d_2)v_i}{2d_2}, B_{i3}^*(v_i) = \frac{v_i}{2}, B_{i2}^*(v_i) = \frac{d_1P_2 - (d_1 - d_2)v_i}{2d_2}, \\
 V_1 &= \frac{P_1 - d_1P_2}{1 - d_1}, V_3 = \frac{P_1 - 2d_2R}{1 - d_2}, V_6 = 2R.
 \end{aligned}$$

We take the derivatives of  $\pi$  in (4.10) with respect to  $P_1, P_2, R$  and set equal to zero, and solve for the optimal solutions as the follows:

$$P_1^* = P_2^* = \frac{2(1 - d_2)}{3 - 4d_2^2}, R^* = \frac{2d_2(1 - d_2)}{3 - 4d_2^2} = d_2P_1^* \tag{4.11}$$

Furthermore, one can show that the Hessian matrix is negative-definite. Substituting optimal prices  $P_1^*, P_2^*$  and  $R^*$  into (4.10), we have the maximum expected revenue  $\pi^*$ ,

$$\pi^* = \frac{1 - d_2}{3 - 4d_2^2} \tag{4.12}$$

We can see that although there are three channels and four customer segments in this scenario, the service provider only has two sources of revenue: REG and BID as the OPQ channel is not generating sales. This is because the price on OPQ is set the same as that on REG and the threshold on the BID channel is set relatively low so that all the consumers in the segment of BID then OPQ will win their bids and will not switch to OPQ.

Parameters  $d_1, d_2$  need to satisfy constraints obtained by substituting optimal prices  $P_1^*, P_2^*$  and  $R^*$  as shown in (4.11) into the conditions in this scenario:  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq -2d_2R(1 - d_1)$ ,  $P_1 - 2d_2R \leq 1 - d_2$ , and  $P_1 \in [0, 1]$ ,  $P_2 \in [0, 1]$ . Here, the constraint is just simply  $0 \leq d_2 \leq 1/2$ . Under this constraint, one can show that both the optimal full information price and the maximum expected revenue in this scenario are more than in the situation where there is only the REG channel, which are  $\frac{1}{2}$  and  $\frac{1}{4}$  respectively. Please see the Appendix for the detailed derivation of the results discussed above.

*Case I - Scenario II.*  $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$  and  $P_2 \geq 2R$ .

As in the previous scenario, the consumers in the first segment  $[V_2, 1]$  (if  $V_2 < 1$ ) buy on REG directly. It is easy to see that  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$  is equivalent to

$$\frac{P_1 - 2d_2R}{1 - d_2} < \frac{P_1 - d_1P_2}{1 - d_1}, \text{ i.e. } V_3 < V_1.$$

Thus, all the consumers in the segment of BID then REG ( $[V_1, \min(V_2, 1))$ ) lose their bids and go to purchase at REG as their bids are below the threshold  $R$  if their valuation is more than  $V_3$ .

One can easily show that  $V_3 < V_1$  implies  $V_5 < V_1$  and if  $P_2 \geq 2R$ , then  $P_2 \leq V_5$ , so the segment of BID then OPQ ( $v_i \in [P_2, \min(V_1, 1))$ ) consists of two groups of

consumers. The first group of consumers with valuations  $v_i \in [V_5, \min(V_1, 1))$  purchase on OPQ and second group  $v_i \in [P_2, \min(V_5, 1))$  win their bids.

$P_2 \geq 2R$  indicates  $P_2 \geq V_6$ , then in the segment of bidding only, consumers with valuations  $v_i \in [V_6, P_2)$  win their bid and consumers with valuations  $v_i \in [0, V_6)$  lose. The revenue for the service provider in this scenario is summarized in Table 4.5.

Table 4.5: Revenue scenarios, Case I - Scenario II

Valuation ( $v_i$ )	Transaction channel	Critical Points		
		$1 > V_1$	$V_1 \geq 1 > V_5$	$V_5 \geq 1$
$[V_1, 1]$	REG ( $P_1$ )	Present	Absent	Absent
$[V_5, \min(V_1, 1))$	OPQ ( $P_2$ )	Present	Present	Absent
$[P_2, \min(V_5, 1))$	BID ( $B_{i2}^*(v_i)$ )	Present	Present	Present
$[V_6, P_2)$	BID ( $B_{i3}^*(v_i)$ )	Present	Present	Present

Thus, if  $1 \geq V_1 > V_3$  and  $1 \geq V_5 \geq P_2$  i.e.  $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ ,  $P_1 - d_1P_2 \leq 1 - d_1$  and  $P_2 \geq 2R$ , then the expected revenue  $\pi$  for the service provider in this scenario is:

$$\begin{aligned}
\pi &= \int_{V_1}^1 P_1 f(v_i) dv_i + \int_{V_5}^{V_1} P_2 f(v_i) dv_i + \int_{P_2}^{V_5} B_{i2}^*(v_i) f(v_i) dv_i + \int_{V_6}^{P_2} B_{i3}^*(v_i) f(v_i) dv_i \\
&= \frac{[4d_2(P_1 - P_1^2 + P_1P_2 - 2P_2R) + d_1^2(-4P_1(-1 + P_2) + P_2^2 - 4R^2) \\
&\quad + \frac{d_1(4P_1^2 + 4P_1(-1 + d_2(-1 + P_2) - P_2) + (3 - 4d_2)P_2^2 + 8d_2P_2R + 4R^2)}{4(-1 + d_1)(d_1 - d_2)}]}{4(-1 + d_1)(d_1 - d_2)} \quad (4.13)
\end{aligned}$$

where, recall that

$$\begin{aligned}
B_{i3}^*(v_i) &= \frac{d_2 v_i}{2}, B_{i2}^*(v_i) = \frac{P_2 - (d_1 - d_2)v_i}{2}, \\
V_1 &= \frac{P_1 - d_1 P_2}{1 - d_1}, V_5 = \frac{d_1 P_2 - 2d_2 R}{d_1 - d_2}, V_6 = 2R.
\end{aligned}$$

As earlier, taking the derivatives of  $\pi$  in (4.13) with respect to  $P_1, P_2, R$  and setting to zero, we solve for the optimal solutions.

$$\begin{aligned} P_1^* &= \frac{d_1^3 + d_1^2(3 - 4d_2) - 4d_2^2 + 4d_1d_2^2}{2(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} \\ P_2^* &= \frac{d_1(1 + d_1)(d_1 - d_2)}{d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2}, \\ R^* &= \frac{d_2(1 + d_1)(d_1 - d_2)}{d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2} \end{aligned} \quad (4.14)$$

One can show that the Hessian matrix is negative-definite and substituting optimal prices  $P_1^*, P_2^*$  and  $R^*$  in (4.13), one can get the maximum expected revenue  $\pi^*$ ,

$$\pi^* = \frac{d_1^3 + d_1^2(3 - 4d_2) - 4d_2^2 + 4d_1d_2^2}{4(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} \quad (4.15)$$

As before parameters  $d_1, d_2$  need to satisfy a set of constraints; which are obtained by substituting optimal prices  $P_1^*, P_2^*$  and  $R^*$  as shown in (4.14) into:  $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1), P_1 - d_1P_2 \leq 1 - d_1, P_2 \geq 2R$ , and  $P_1 \in [0, 1], P_2 \in [0, 1]$ .

Hence, the constraints that  $d_1, d_2$  need to satisfy are shown in (4.16), (4.17), (4.18), and (4.19). Similar to the previous scenario, under these constraints, one can show that both the optimal full information price and the maximum expected revenue in this scenario are larger than  $\frac{1}{2}$  and  $\frac{1}{4}$  respectively. The appendix provides the detailed derivation.

$$d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2 \geq 0 \quad (4.16)$$

$$d_2 \leq \frac{d_1}{2} \quad (4.17)$$

$$4d_2^3 + d_1^2(-1 + 2d_2) + d_1(d_2 - 6d_2^2) < 0 \quad (4.18)$$

$$4d_1d_2^2 - 4d_2^3 + d_1^3(-1 + 2d_2) + d_1^2(d_2 - 2d_2^2) \geq 0 \quad (4.19)$$

Case I - Scenario III.  $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$  and  $P_2 < 2R$ .

As in previous scenarios, consumers in the first segment  $[V_2, 1]$  (if  $V_2 < 1$ ) buy on REG directly. And  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$  implies

$$\frac{P_1 - 2d_2R}{1 - d_2} < \frac{P_1 - d_1P_2}{1 - d_1}, \text{ i.e. } V_3 < V_1.$$

Hence, all the consumers in the segment BID then REG ( $[V_1, \min(V_2, 1))$ ) lose their bids and purchase at the REG channel as their bids are below the threshold  $R$  if their valuation is more than  $V_3$ .

If  $P_2 < 2R$ , then  $V_5 < P_2$ , so all consumers in the segment of BID then OPQ lose their bids and switch back to the OPQ channel to buy as their bids are less than the threshold  $R$  if the valuation  $v_i > V_5$ .

$P_2 < 2R$  also indicates  $P_2 < V_6$ , then in the segment of bidding only, all consumers will lose their bid as  $B_i < R$  if their valuation is less than  $V_6$ . Table 4.6 summarizes these revenue scenarios.

Table 4.6: Revenue scenarios, Case I - Scenario III

Valuation ( $v_i$ )	Transaction channel	Critical Points	
		$1 > V_1$	$V_1 \geq 1 > P_2$
$[V_1, 1]$	REG ( $P_1$ )	Present	Absent
$[P_2, \min(V_1, 1))$	OPQ ( $P_2$ )	Present	Present

Therefore, if  $1 \geq V_1$  and  $P_2 \geq V_5$  i.e.  $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ ,  $P_1 - d_1P_2 \leq 1 - d_1$  and  $P_2 < 2R$ , then every possible transaction interval in

Table 4.6 is present and produces  $\pi$ :

$$\begin{aligned}\pi &= \int_{V_1}^1 P_1 f(v_i) dv_i + \int_{P_2}^{V_1} P_2 f(v_i) dv_i \\ &= \frac{(1 - d_1 - P_1 + d_1 P_2)P_1 + P_2(P_1 - P_2)}{1 - d_1}\end{aligned}\quad (4.20)$$

Taking the first partial derivatives of  $\pi$  with respect to  $P_1$  and  $P_2$  respectively and setting them to zero,

$$P_1^* = \frac{2}{3 + d_1}, P_2^* = \frac{1 + d_1}{3 + d_1} \quad (4.21)$$

It is easy to show that the Hessian matrix is again negative-definite, substituting  $P_1^*$  and  $P_2^*$  in (4.21) into (4.20), and we have the maximum expected revenue  $\pi^*$  is:

$$\pi^* = \frac{1}{3 + d_1} \quad (4.22)$$

As shown above REG and OPQ are the only two channels with sales. This happens when the threshold on the BID channel is set so high that the consumers in both BID then OPQ segment and the BID only segment lose their bid. In fact, from the conditions  $0 \geq (d_1 - d_2)P_1 - d_1 P_2(1 - d_2) > -2d_2 R(1 - d_1)$ ,  $P_1 - d_1 P_2 \leq 1 - d_1$  and  $P_2 < 2R$  we can see that

$$R^* > \max \left\{ \frac{P_2^*}{2}, \frac{-(d_1 - d_2)P_1^* + d_1 P_2^*(1 - d_2)}{2d_2(1 - d_1)} \right\}.$$

By substituting  $P_1^*$  and  $P_2^*$  shown in (4.21) back into the inequality above, we have

$$R^* > \max \left\{ \frac{1 + d_1}{2(3 + d_1)}, \frac{2d_2 - d_1(1 - d_2)}{2d_2(3 + d_1)} \right\}.$$

Since

$$\frac{2d_2 - d_1(1 - d_2)}{2d_2(3 + d_1)} - \frac{1 + d_1}{2(3 + d_1)} = \frac{d_2 - d_1}{6d_2 + 2d_1d_2},$$

so the lower bound of the optimal threshold

$$R_L^* = \begin{cases} \frac{2d_2 - d_1(1-d_2)}{2d_2(3+d_1)} & \text{if } d_1 < d_2 \\ \frac{1+d_1}{2(3+d_1)} & \text{otherwise} \end{cases} \quad (4.23)$$

Therefore, the optimal threshold  $R^* \in [R_L^*, P_2^*]$ . Substituting the optimal solutions  $P_1^*, P_2^*$ , and  $R^* > P_2^*/2$  back in the conditions  $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ ,  $P_1 - d_1P_2 \leq 1 - d_1$  and  $P_2 < 2R$  we get the constraint that  $d_1$  and  $d_2$  need to satisfy  $d_1 \leq 2d_2/(1 - d_2)$ . It is easy to see that  $\leq 2d_2/(1 - d_2) > d_2$ , thus, the lower bound of the optimal threshold becomes

$$R_L^* = \begin{cases} \frac{2d_2 - d_1(1-d_2)}{2d_2(3+d_1)} & \text{if } d_1 < d_2 \\ \frac{1+d_1}{2(3+d_1)} & \text{if } d_2 \leq d_1 \leq \frac{2d_2}{1-d_2} \end{cases} \quad (4.24)$$

One can check that under this constraint the optimal REG price and the maximum expected revenue in this scenario are greater than or equal to those values in the situation where there is only REG channel i.e.  $\frac{1}{2}$  and  $\frac{1}{4}$  respectively. The details are provided in the Appendix.

**Case II.**  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0 \Leftrightarrow V_4 < V_2 < V_1$ .

The market segmentation in Case II was previously summarized in Table 4.3. Note that this case only exists when  $d_1 > d_2$ , since when  $d_1 \leq d_2$ ,  $d_1P_2(1 - d_2) - (d_1 - d_2)P_1 > 0$ .

$V_4 = \frac{d_1P_2}{d_1 - d_2}$ ,  $V_5 = \frac{P_2 - 2R}{d_1 - d_2}$  implies  $V_4 \geq P_2$ , and  $V_4 \geq V_5$ . Recall that the consumers with valuations  $v_i < V_5$  and  $v_i > V_6$  will win their bid. Hence, we only need to compare the critical points  $V_5$ ,  $P_2$  and  $V_6$  to determine revenue, resulting in two revenue scenarios based on  $d_1, d_2, P_1, P_2$ , and  $R$  as discussed in the following .

*Case II - Scenario I.*  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$  and  $2R \leq P_2$ .

Recall that the segment  $[V_1, 1]$  (if  $V_1 < 1$ ) and  $[V_4, \min(V_1, 1))$  are the segment of REG only and OPQ only segments respectively. If  $2R \leq P_2$ , then  $V_5 \geq P_2 \geq V_6$ , then the segment of BID then OPQ ( $v_i \in [P_2, \min(V_4, 1))$ ) consists of two groups of consumers. The first group of consumers with valuation  $v_i \in [V_5, \min(V_4, 1))$  purchases on OPQ and second group  $v_i \in [P_2, \min(V_5, 1))$  have winning bids. And in the segment of bidding only, consumers with valuation  $v_i \in [V_6, P_2)$  win their bid and consumers with valuation  $v_i \in [0, V_6)$  lose. Table 4.7 summarizes revenue scenarios.

Table 4.7: Revenue segmentation, Case II - Scenario I

Valuation ( $v_i$ )	Transaction channel	Critical Points		
		$1 > V_1$	$V_1 \geq 1 > V_5$	$V_5 \geq 1$
$[V_1, 1]$	REG ( $P_1$ )	Present	Absent	Absent
$[V_5, \min(V_1, 1))$	OPQ ( $P_2$ )	Present	Present	Absent
$[P_2, \min(V_5, 1))$	BID ( $B_{i2}^*(v_i)$ )	Present	Present	Present
$[V_6, P_2)$	BID ( $B_{i3}^*(v_i)$ )	Present	Present	Present

Thus, if  $V_1 \leq 1$  and  $V_5 \geq P_2 \geq V_6$  i.e.  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$  and  $P_1 - d_1P_2 \leq 1 - d_1$  and  $2R \leq P_2$ , then every possible transaction interval in Table 4.7 is present, with  $\pi$  for the service provider in this scenario:

$$\pi = \int_{V_1}^1 P_1 f(v_i) dv_i + \int_{V_5}^{V_1} P_2 f(v_i) dv_i + \int_{P_2}^{V_5} B_{i2}^*(v_i) f(v_i) dv_i + \int_{V_6}^{P_2} B_{i3}^*(v_i) f(v_i) dv_i \quad (4.25)$$

where, recall that

$$B_{i3}^*(v_i) = \frac{d_2 v_i}{2}, B_{i2}^*(v_i) = \frac{P_2 - (d_1 - d_2)v_i}{2},$$

$$V_1 = \frac{P_1 - d_1 P_2}{1 - d_1}, V_5 = \frac{d_1 P_2 - 2d_2 R}{d_1 - d_2}, V_6 = 2R.$$



As in Case I, Scenario II above the service provider uses all three channels by setting the appropriate threshold and prices on the channels. And it has the same revenue expression (4.25) but with slightly different parameter constraints. Therefore, the optimal prices and expected revenue in this scenario are also given by (4.14), (4.15) above respectively. One can derive the parameter constraints that need to be satisfied in this scenario by substituting  $P_1^*, P_2^*$  and  $R^*$  in (4.14) into conditions  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0, P_1 - d_1P_2 \leq 1 - d_1, 2R \leq P_2, P_1 \in [0, 1], P_2 \in [0, 1]$ , and  $P_2 \leq P_1$ .

Specifically, the constraints are (4.16), (4.17), (4.18), and

$$4d_1d_2^2 - 4d_2^3 + d_1^3(-1 + 2d_2) + d_1^2(d_2 - 2d_2^2) < 0, \quad (4.26)$$

whose inequality sign is the opposite of the fourth condition (4.19) in Case I, Scenario II.

*Case II - Scenario II.*  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$  and  $P_1 - d_1P_2 \leq 1 - d_1$  and  $2R > P_2$ .

Consumers in the segment of  $[V_1, 1]$  (if  $V_1 < 1$ ) and  $[V_4, \min(V_1, 1))$  buy at REG only and OPQ only respectively. If  $2R > P_2$ , then  $V_5 < P_2 < V_6$ , for both the segments of BID then OPQ ( $v_i \in [P_2, \min(V_4, 1))$ ) and BID only ( $v_i \in [0, P_2)$ ), there are no consumers with winning bids.

Table 4.8: Revenue segmentation, Case II - Scenario II

Valuation ( $v_i$ )	Transaction channel	Critical Points	
		$1 > V_1$	$V_1 \geq 1 > P_2$
$[V_1, 1]$	REG ( $P_1$ )	Present	Absent
$[P_2, \min(V_1, 1))$	OPQ ( $P_2$ )	Present	Present

Therefore, with  $1 \geq V_1$  and  $P_2 \geq V_5$  i.e.  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$  and  $P_1 - d_1P_2 \leq 1 - d_1$  and  $2R > P_2$  the resulting expected revenue  $\pi$ :

$$\pi = \int_{V_1}^1 P_1 f(v_i) dv_i + \int_{P_2}^{V_1} P_2 f(v_i) dv_i \quad (4.27)$$

Substituting optimal prices  $P_1^*$  and  $P_2^*$ , as shown in 4.21 and  $R^* > P_2^*/2$  back in the conditions  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$  and  $P_1 - d_1P_2 \leq 1 - d_1$  and  $2R > P_2$  and  $P_2 < 2R$  we get  $d_1 > 2d_2/(1 - d_2)$  as the parameter constraints in scenario. Note that this is just with an opposite sign from the parameter condition in Case I - Scenario III. As  $R^* > P_2^*/2$ , we have the lower bound of the optimal threshold

$$R_L^* = P_2^*/2 = \frac{1 + d_1}{2(3 + d_1)},$$

and the optimal threshold will still be  $R^* \in [R_L^*, P_2^*]$ .

Overall, in the situation where the service provider releases their products on all three channels: REG, OPQ and BID, we have five scenarios of revenue generation as summarized in Table 4.9.

Table 4.9: Revenues, prices and bidding threshold summaries

Scenario	Optimal price on REG ( $P_1^*$ )	Optimal price on OPQ ( $P_2^*$ )	Optimal threshold on BID ( $R^*$ )	Maximum expected revenue ( $\pi^*$ )	Parameter conditions ( $d_1, d_2$ )
Case I-I	$\frac{2(1-d_2)}{3-4d_2^2}$	$\frac{2(1-d_2)}{3-4d_2^2}$	$\frac{2d_2(1-d_2)}{3-4d_2^2}$	$\frac{1-d_2}{3-4d_2^2}$	$0 \leq d_2 \leq \frac{1}{2}$
Case I-II	$\frac{d_1^3 + d_1^2(3-4d_2) - 4d_2^2 + 4d_1d_2^2}{2(d_1^3 - d_1^2(-2+d_2) + d_1d_2 - 4d_2^2)}$	$\frac{d_1(1+d_1)(d_1-d_2)}{d_1^3 - d_1^2(-2+d_2) + d_1d_2 - 4d_2^2}$	$\frac{d_2(1+d_1)(d_1-d_2)}{d_1^3 - d_1^2(-2+d_2) + d_1d_2 - 4d_2^2}$	$\frac{d_1^3 + d_1^2(3-4d_2) - 4d_2^2 + 4d_1d_2^2}{4(d_1^3 - d_1^2(-2+d_2) + d_1d_2 - 4d_2^2)}$	(4.16), (4.17), (4.18), (4.19)
Case I-III	$\frac{2}{3+d_1}$	$\frac{1+d_1}{3+d_1}$	$\left[ \frac{2d_2 - d_1(1-d_2)}{2d_2(3+d_1)}, \frac{1+d_1}{3+d_1} \right]$	$\frac{1}{3+d_1}$	$d_1 < d_2$
	$\frac{2}{3+d_1}$	$\frac{1+d_1}{3+d_1}$	$\left[ \frac{1+d_1}{2(3+d_1)}, \frac{1+d_1}{3+d_1} \right]$	$\frac{1}{3+d_1}$	$d_2 \leq d_1 \leq \frac{2d_2}{1-d_2}$
Case II-I	Same as Case I-II				(4.16), (4.17), (4.18), (4.26)
Case II-II	$\frac{2}{3+d_1}$	$\frac{1+d_1}{3+d_1}$	$\left[ \frac{1+d_1}{2(3+d_1)}, \frac{1+d_1}{3+d_1} \right]$	$\frac{1}{3+d_1}$	$d_1 > \frac{2d_2}{1-d_2}$

## Illustration of the Optimal Policies

In this section we illustrate optimal policies and the resulting segmentation substituting different values of  $d_1$  and  $d_2$  into the closed-form solutions discussed previously and plot them to illustrate the impact of channel opacity on the revenue, prices, and threshold.

We plot the maximum expected revenue that the service provider can obtain while releasing products on REG, OPQ and BID channels and the corresponding optimal prices and threshold set on those channels as given in (4.9). It is analytically illustrated in the Appendix that the maximum expected revenue and the posted price on REG in all five scenarios are more than those in the base case: REG only, so the maximum expected revenue and corresponding posted price on REG in this three channel case are also greater than those values in the REG only case ( $\frac{1}{4}$  and  $\frac{1}{2}$  respectively). One can also see this property in the plots. Figure 4.9 plots optimal expected revenues resulting from optimal full information prices (Figure 4.10), optimal opaque prices (Figure 4.11) and optimal bidding thresholds (Figure 4.12).

As shown in Figures 4.10, 4.11 prices on REG decrease as BID and OPQ become less opaque, conversely OPQ prices increase (and converge to REG prices) as opacity on OPQ decreases, but decreases as opacity on BID decreases. Figure 4.12 shows that BID thresholds increase as opacity on BID decreases, but decreases as opacity on OPQ decreases. The impacts of channel opacity on optimal prices and threshold indicate that when the products on the opaque channels (OPQ and BID) become more valuable (less opaque), the price on the full information channel (REG) can not be set too high to lose consumers. Similarly, if the BID channel becomes less opaque, some consumers on OPQ may switch

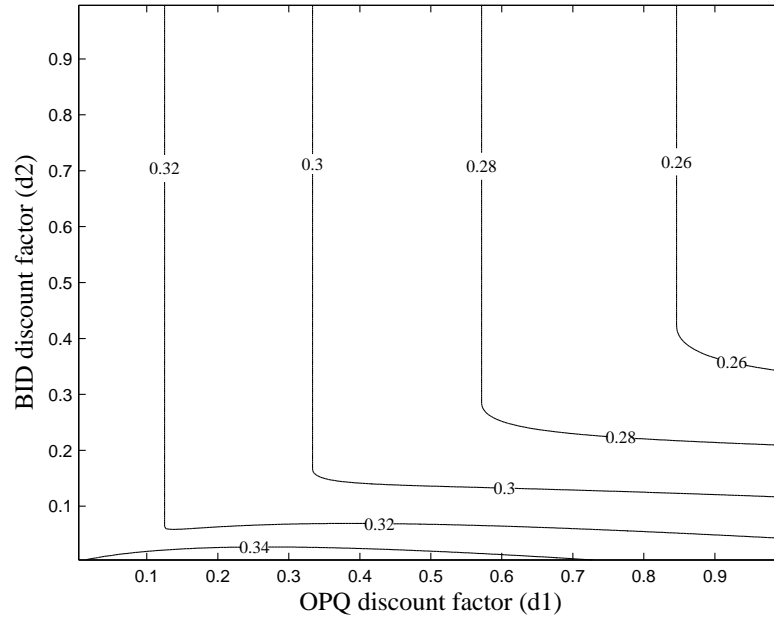


Figure 4.9: Optimal expected revenue

to bid on the BID channel due to a potential chance of getting a better value with less price.

As displayed in Figure 4.9 the expected revenue decreases when either OPQ or BID channel opacity degree decreases ( $d$ 's increase). This implies that the more opaque those opaque channels are, the more segments in the market and so the service provider can capture more consumers because of their heterogeneous valuations of the product.

### 4.2.3 Optimal Service Provider Policies with Limited Capacity

In this section, we consider a service provider with limited capacity and as such the firm would logically limit sales at lower prices. We assume that the service

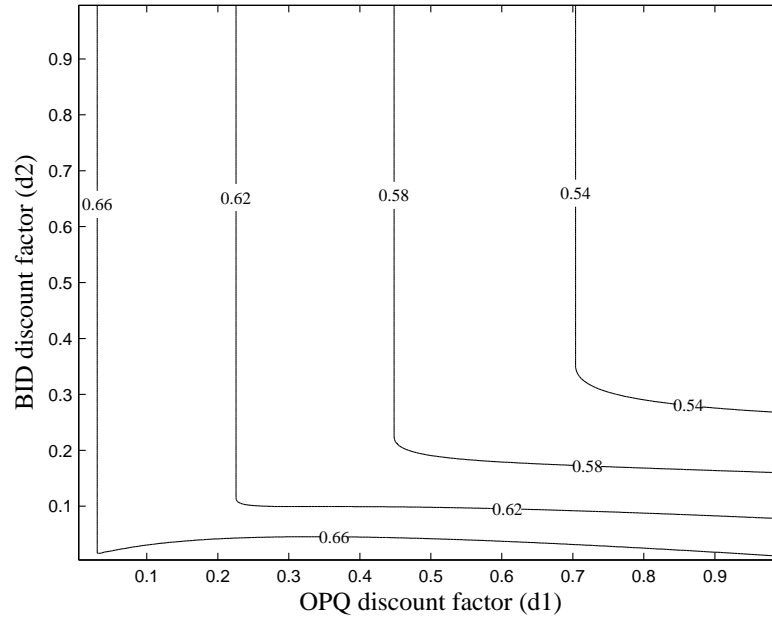


Figure 4.10: Optimal regular prices

provider has a limited inventory of capacity  $C < 1$ . Although we have the capacity constraint in this case, the consumer segmentation and the revenue segmentation will still be the same as the case with abundant capacity discussed in earlier sections. However, the effective segments (those which generate sales), the optimal pricing policy and the maximum expected revenue that the service provider can achieve will depend on capacity.

As the firm is simultaneously using all three channels we assume the customer segments arrive in a random order, i.e. first come first serve. Thus, the capacity  $C$  will be allocated to each segment of the market proportional to its size relative to the total demand in the market, otherwise referred to as random or proportionate splitting.

As an illustration on how the limited capacity influences the service

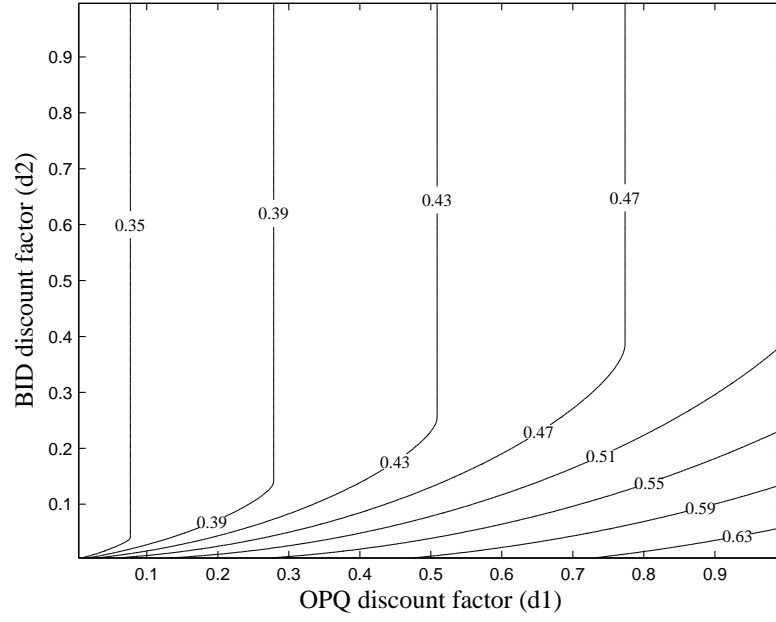


Figure 4.11: Optimal opaque prices

provider's pricing policy and maximum revenue that can be achieved, we assume that the service provider sells the products only on the REG channel and set its price as  $P_1$ .

Similar to the situation with no capacity constraint, consumers with valuation higher than the price  $P_1$  will purchase through this channel. However, demand can be met only up to  $C$ . In other words, when the total demand  $1 - P_1 \leq C$ , the situation is exactly the same as the case with no capacity constraint discussed previously, and so the revenue is  $\pi_C = (1 - P_1)P_1$ , which reaches the maximum value  $\pi_C^* = \frac{1}{4}$  at  $P_{1C}^* = \frac{1}{2}$ . But when  $1 - P_1 > C$ , the revenue is  $\pi_C = CP_1$ . Thus, the price  $P_1$  increases until it reaches the upper bound  $1 - C$  and the maximum revenue  $\pi_C^* = C(1 - C)$  is achieved.

We summarize the results as the following:

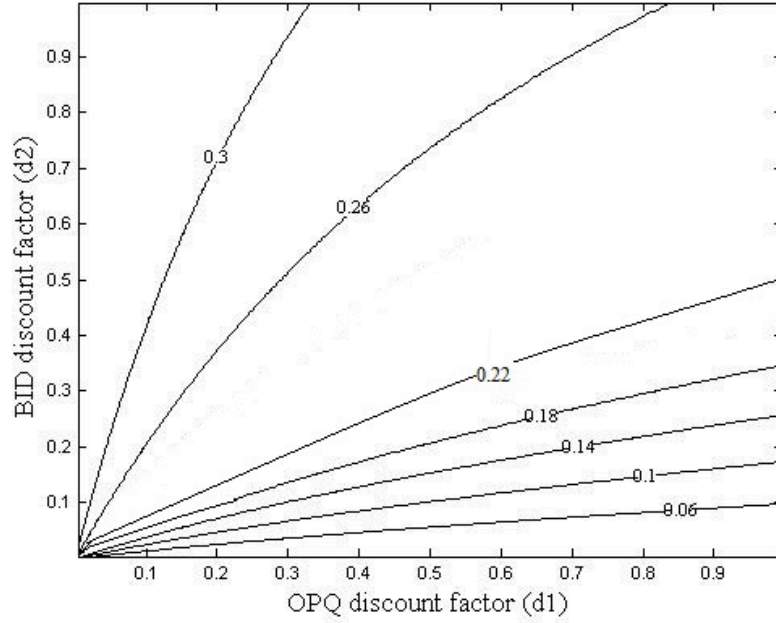


Figure 4.12: Optimal bidding thresholds

If  $1 - C \leq \frac{1}{2}$  i.e.  $C \geq \frac{1}{2}$ , then  $\pi_C^* = \frac{1}{4}$  and  $P_{1C}^* = \frac{1}{2}$ ;

If  $1 - C > \frac{1}{2}$  i.e.  $C < \frac{1}{2}$ , then  $\pi_C^* = C(1 - C)$  and  $P_{1C}^* = 1 - C$ .

One can easily show that when  $C < \frac{1}{2}$ ,  $\pi_C^* = C(1 - C)$  is an increasing function in the capacity  $C$ , and  $P_{1C}^*$  is decreasing in  $C$ . These are quite intuitive as when we can not meet all demand we receive less revenue but through higher prices.

The service provider now lists the products on the REG, OPQ and BID simultaneously. They set price  $P_1$  on REG,  $P_2$  on OPQ, a bidding threshold  $R$  on BID.

#### Constrained Case I.

Capacity  $C$  is allocated proportionally and from (4.10) we know that the total

demand that is supposed to be met if we have enough capacity is  $1 - 2R$ . Thus, if the conditions  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq -2d_2R(1 - d_1)$ ,  $P_1 - 2d_2R \leq 1 - d_2$ , are satisfied and  $C \geq 1 - 2R$ , then the expected revenue  $\pi_C$  is the same as the revenue  $\pi$  in the unconstrained case. If  $C < 1 - 2R$ , then the expected revenue  $\pi_C$  is :

$$\begin{aligned}
\pi_C &= \frac{\int_{V_3}^1 f(v_i)dv_i}{1 - 2R} \cdot C \cdot P_1 + \frac{\int_{V_1}^{V_3} f(v_i)dv_i}{1 - 2R} \cdot C \cdot \frac{\int_{V_1}^{V_3} B_{i1}^*(v_i)f(v_i)dv_i}{\int_{V_1}^{V_3} f(v_i)dv_i} \\
&\quad + \frac{\int_{P_2}^{V_1} B_{i2}^*(v_i)f(v_i)dv_i}{1 - 2R} \cdot C \cdot \frac{\int_{P_2}^{V_1} B_{i2}^*(v_i)f(v_i)dv_i}{\int_{P_2}^{V_1} f(v_i)dv_i} \\
&\quad + \frac{\int_{V_6}^{P_2} B_{i3}^*(v_i)f(v_i)dv_i}{1 - 2R} \cdot C \cdot \frac{\int_{V_6}^{P_2} B_{i3}^*(v_i)f(v_i)dv_i}{\int_{V_6}^{P_2} f(v_i)dv_i} \\
&= \frac{C}{1 - 2R} \left[ \int_{V_3}^1 P_1 f(v_i)dv_i + \int_{V_1}^{V_3} B_{i1}^*(v_i)f(v_i)dv_i + \int_{P_2}^{V_1} B_{i2}^*(v_i)f(v_i)dv_i \right. \\
&\quad \left. + \int_{V_6}^{P_2} B_{i3}^*(v_i)f(v_i)dv_i \right] \\
&= \frac{C}{1 - 2R} \cdot \pi
\end{aligned} \tag{4.28}$$

where,

$$\begin{aligned}
B_{i1}^*(v_i) &= \frac{P_1 - (1 - d_2)v_i}{2d_2}, B_{i3}^*(v_i) = \frac{v_i}{2}, B_{i2}^*(v_i) = \frac{d_1P_2 - (d_1 - d_2)v_i}{2d_2}, \\
V_1 &= \frac{P_1 - d_1P_2}{1 - d_1}, V_3 = \frac{P_1 - 2d_2R}{1 - d_2}, V_6 = 2R,
\end{aligned}$$

$\pi$  = the revenue of the Case I-Scenario I without a capacity constraint.

### Constrained Case II

Case I - Scenario II and Case II - Scenario I have the same revenue functions in terms of  $P_1, P_2, R, d_1$  and  $d_2$  as shown in (4.13), but with different parameter constraints. Thus, in the case with capacity constraint  $C$  we combine these two scenarios together, and similar to the Constrained Case I above, the total demand that we need to meet if we have abundant capacity is  $1 - 2R$ .



If the conditions of the scenarios  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ ,  $P_1 - d_1P_2 \leq 1 - d_1$ ,  $P_2 \geq 2R$ , are satisfied and  $C \geq 1 - 2R$ , the expected revenue  $\pi_C$  is the same as the revenue  $\pi$  in the case of no capacity constraint. If  $C < 1 - 2R$ , the expected revenue  $\pi_C$  with capacity  $C$  is given as below:

$$\begin{aligned}
\pi_C &= \frac{\int_{V_1}^1 f(v_i)dv_i}{1 - 2R} \cdot C \cdot P_1 + \frac{\int_{V_5}^{V_1} f(v_i)dv_i}{1 - 2R} \cdot C \cdot P_2 + \frac{\int_{P_2}^{V_5} f(v_i)dv_i}{1 - 2R} \cdot C \cdot \frac{\int_{P_2}^{V_5} B_{i2}^*(v_i)f(v_i)dv_i}{\int_{P_2}^{V_5} f(v_i)dv_i} \\
&\quad + \frac{\int_{V_6}^{P_2} f(v_i)dv_i}{1 - 2R} \cdot C \cdot \frac{\int_{V_6}^{P_2} B_{i3}^*(v_i)f(v_i)dv_i}{\int_{V_6}^{P_2} f(v_i)dv_i} \\
&= \frac{C}{1 - 2R} \left[ \int_{V_1}^1 P_1 f(v_i)dv_i + \int_{V_5}^{V_1} P_2 f(v_i)dv_i + \int_{P_2}^{V_5} B_{i2}^*(v_i)f(v_i)dv_i \right. \\
&\quad \left. + \int_{V_6}^{P_2} B_{i3}^*(v_i)f(v_i)dv_i \right] \\
&= \frac{C}{1 - 2R} \cdot \pi
\end{aligned} \tag{4.29}$$

where,

$$\begin{aligned}
B_{i3}^*(v_i) &= \frac{d_2 v_i}{2}, B_{i2}^*(v_i) = \frac{P_2 - (d_1 - d_2)v_i}{2}, \\
V_1 &= \frac{P_1 - d_1 P_2}{1 - d_1}, V_5 = \frac{d_1 P_2 - 2d_2 R}{d_1 - d_2}, V_6 = 2R,
\end{aligned}$$

$\pi$  = the revenue of the Case II-Scenario I or Case II-Scenario I in the case with no capacity constraint.

### Constrained Case III.

Recall that Case I-Scenario III and Case II-Scenario II have the same revenue functions of  $P_1, P_2, R, d_1$  and  $d_2$  as given in (4.20) but with different parameter constraints. Hence, we combine these two scenarios together in the setting with constrained capacity and note the total demand that we need to meet if we have abundant capacity is now  $1 - P_2$ .

Thus, if the conditions  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ ,  $P_1 - d_1P_2 \leq 1 -$

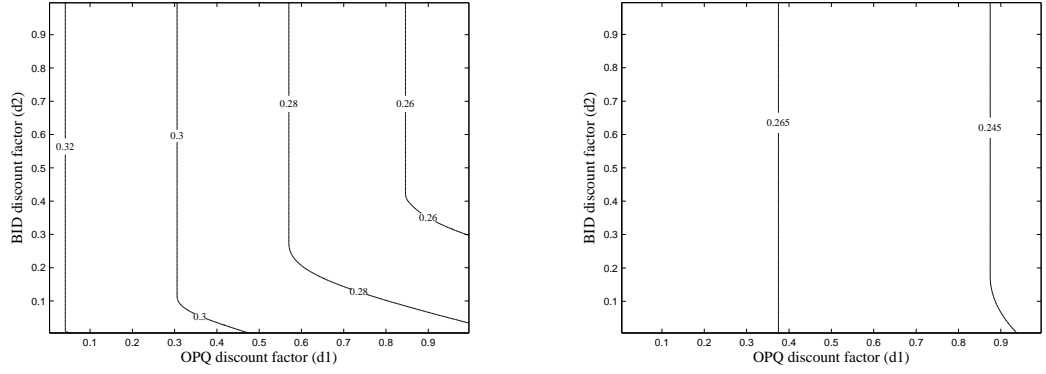


Figure 4.13: Optimal expected revenue-left capacity=0.55, right capacity=0.40

$d_1, P_2 < 2R$  in the scenarios are satisfied and  $C \geq 1 - P_2$ , then the expected revenue  $\pi_C$  with capacity  $C$  is the same as the expected revenue  $\pi$  with no constrained capacity. If  $C < 1 - P_2$ , then the expected revenue  $\pi_C$  given as the follows.

$$\begin{aligned}
 \pi_C &= \frac{\int_{V_1}^1 f(v_i) dv_i}{1 - P_2} \cdot C \cdot P_1 + \frac{\int_{P_2}^{V_1} f(v_i) dv_i}{1 - P_2} \cdot C \cdot P_2 \\
 &= \frac{C}{1 - P_2} \left[ \int_{V_1}^1 P_1 f(v_i) dv_i + \int_{P_2}^{V_1} P_2 f(v_i) dv_i \right] \\
 &= \frac{C}{1 - P_2} \cdot \pi
 \end{aligned} \tag{4.30}$$

where,

$$V_1 = \frac{P_1 - d_1 P_2}{1 - d_1}.$$

$\pi$  = the revenue of the Case I-Scenario II or Case II Scenario II sub-scenario I-ii with no capacity constraint.

We solve for the optimal prices, threshold and the resulting maximum expected revenue. The resulting optimal revenues, prices and thresholds for capacities of 0.55 and 0.40 are shown in Figures 4.13, 4.14, 4.15 and 4.16 respectively.

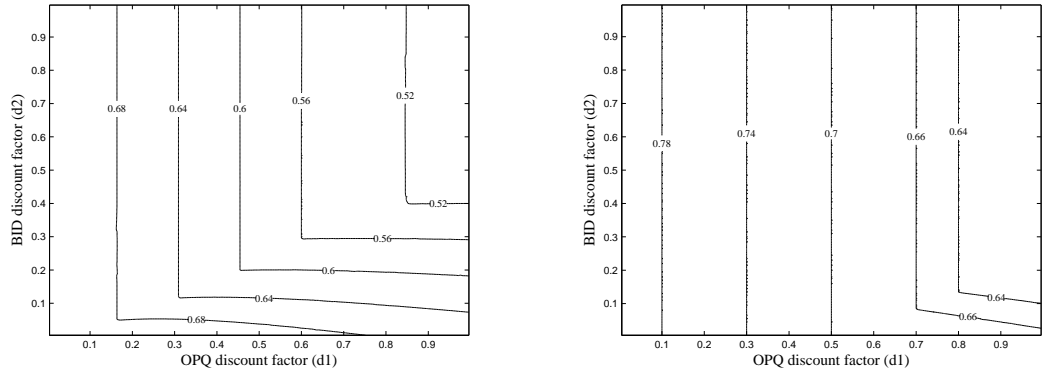


Figure 4.14: Optimal regular prices-left capacity=0.55, right capacity=0.40

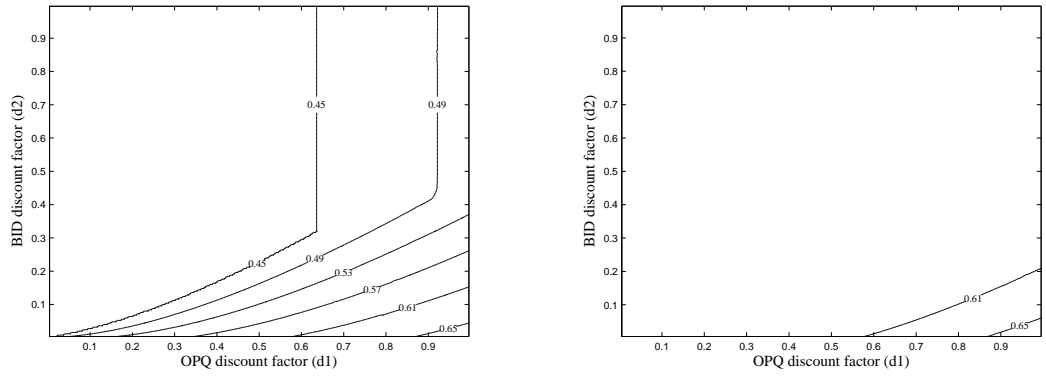


Figure 4.15: Optimal opaque prices - left capacity=0.55, right capacity=0.40

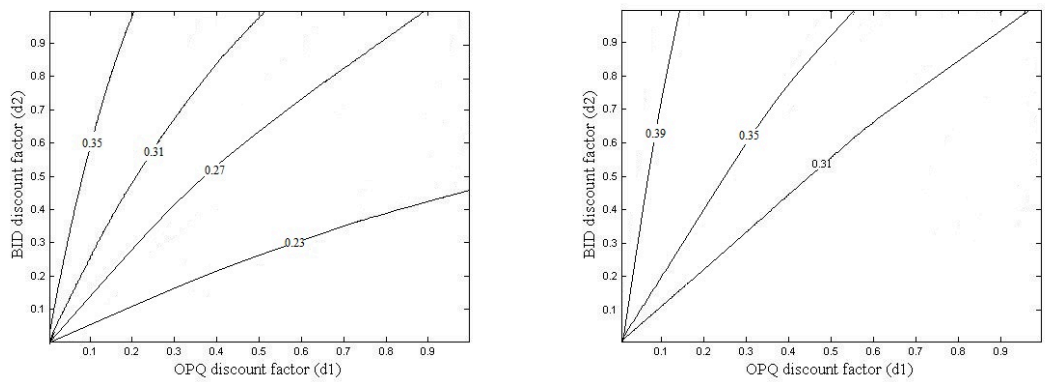


Figure 4.16: Optimal bidding thresholds - left capacity=0.55, right capacity=0.40

### 4.3. Discussion

Figure 4.17 illustrates the the impact of capacity constraints upon expected revenue for a series of opacity levels. As the figure illustrates firms are not overly impacted by capacity limitations until capacity levels below 0.55. With capacity levels between 0.55 and 1 they can capture potential lost revenue opportunities via higher prices. Comparison of Figures 4.14, 4.15 and 4.16 with Figures 4.10, 4.11 and 4.12 shows how firms increase prices and thresholds as capacity decreases. The impacts of these increased prices and thresholds upon segmentation is illustrated in Figure 4.18. The left panels in Figure 4.18 display the channels across which the service provider conducts transactions provided they set optimal prices and thresholds. These transactions are a function of consumer self-selected segmentation as illustrated by the right panels in Figure 4.18.

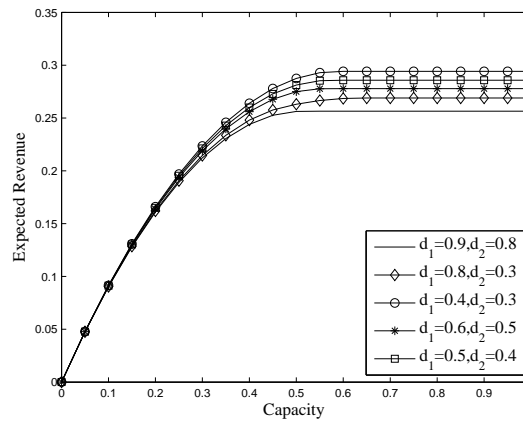
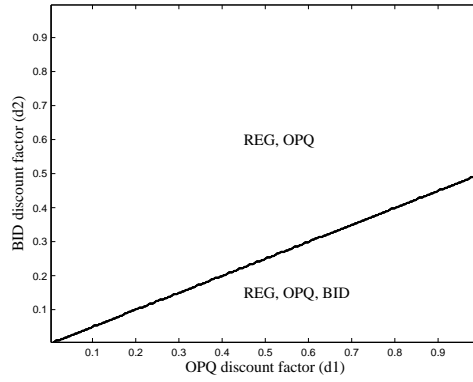
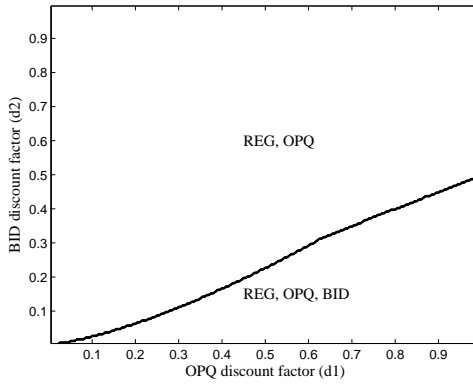
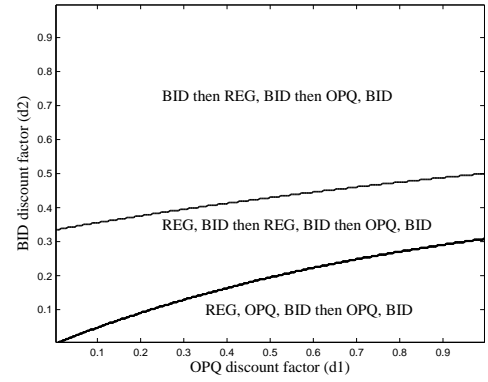


Figure 4.17: Optimal expected revenue as a function of capacity

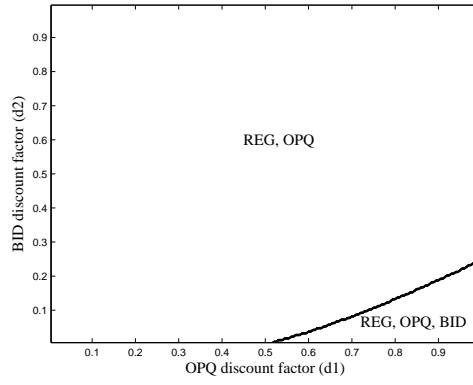
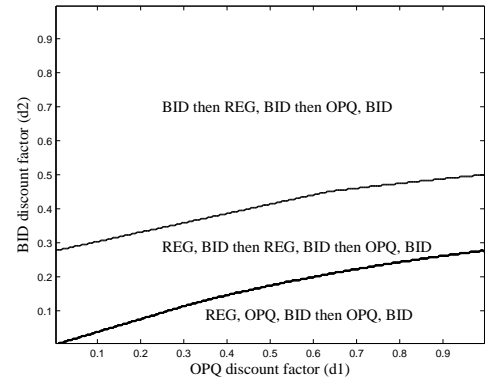
Together these figures illustrate the impacts of opaque selling and under what conditions it appears fruitful to consumers and service providers. Firms should always adopt at least two channels, selling via opaque posted prices



Capacity=1



Capacity=0.55



Capacity=0.40

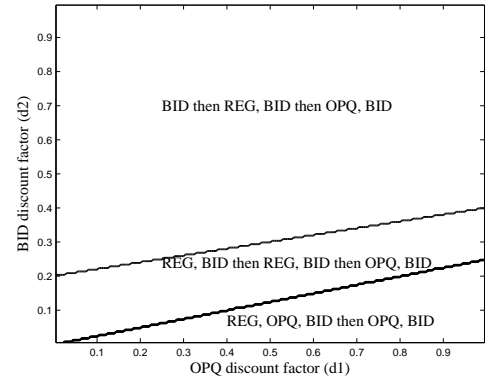


Figure 4.18: Revenue generating channels (left) and consumer segmentation (right)

in addition to regular full information prices. The opaque posted prices simply approach regular full information prices as the opaque channel becomes less opaque - this is consistent across unlimited and constrained capacity settings. Similarly firms should employ opaque bidding but only when opacity of this channel is significant - for example when  $d_2 < \frac{1}{2}$  for capacity of 0.55 with a decreasing desire to use as capacity becomes tightly constrained ( $d_2 < \frac{1}{4}$  for capacity=0.4). It is important to realize that the firm should always be using all three channels, with posted opaque prices/thresholds set too high such that no transactions occur under conditions of decreased opacity. As capacity becomes tighter, the required degree of opacity increases (for continued use of opaque channels) as do prices and thresholds.

As indicated earlier, and as displayed in Figures 4.9 and 4.13 that the maximum expected revenue decreases as opacity decreases. This implies that the more opaque those opaque channels are, the more segments in the market and so the service provider can capture more consumers because of their heterogeneous valuations of the product. This is consistent with what we see in practice as opaque channels tend to separate themselves along degrees of opacity, for example Hotwire.com provides information of hotel amenities as well as feedback from recent guests whereas Priceline.com provides neither on its NYOP bidding channel indicating Hotwire is probably less opaque than Priceline. As a result of this prices on Hotwire are less than those on full information channels but higher than bids typically accepted at Priceline.

In summary we have developed a stylized model of when and how to deploy an opaque selling strategy in concert with regular full information pricing. Unlike previous research which usually assumes an exogenous consumer sepa-

ration into regular consumers and opaque consumers we endogenously model this channel selection process as a function of prices and channel characteristics (opacity). We have shown that even in the face of capacity constraints firms should be simultaneously using opaque channels in concert with regular channels whereas historically focus has been on using opaque channels to sell distressed or otherwise unsellable inventory (surplus capacity). The simultaneous use of opaque selling with regular full information selling effectively segments consumers - allowing firms to sell at higher prices to higher valuation/brand loyal consumers and at lower prices to lower valuation/brand agnostic shoppers via opaque channels and increase firm revenues.

## Appendix

This section includes the detailed discussion and derivation for some of the results in Case I - Scenarios I, II and III in section 4.2.2.

### *Case I - Scenario I.*

Here, we derive the constraints that parameters  $d_1, d_2$  need to satisfy in this sub-scenario by substituting optimal prices  $P_1^*, P_2^*$  and  $R^*$  as shown in (4.11) into the conditions in this scenario:  $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq -2d_2R(1 - d_1), P_1 - 2d_2R \leq 1 - d_2, P_1 \in [0, 1], P_2 \in [0, 1],$ .

$$\begin{aligned}
 0 \leq P_1 \leq 1 &\Rightarrow 0 \leq d_2 \leq \frac{\sqrt{5} + 1}{4}; \\
 (d_1 - d_2)P_1 - d_1P_2(1 - d_2) &\leq -2d_2R(1 - d_1) \\
 \Rightarrow (d_1 - d_2) - d_1(1 - d_2) &\leq -2d_2^2(1 - d_1) \Rightarrow 0 \leq d_2 \leq \frac{1}{2}; \\
 P_1 - 2d_2R &\leq 1 - d_2 \\
 \Rightarrow \frac{2(1 - 2d_2^2)}{3 - 4d_2^2} &\leq 1 \text{ which is true given } 0 \leq d_2 \leq \frac{1}{2} \text{ above.} \quad (4.31)
 \end{aligned}$$

Therefore, we obtain the constraint that the parameter  $d_2$  needs to satisfy, which is  $0 \leq d_2 \leq 1/2$ . Under this constraint, one can show that both the optimal full information price and the maximum expected revenue in this sub-scenario are more than those values in case I where there is only the REG channel, which are  $1/2$  and  $1/4$  respectively, since we have the follows:

$$\begin{aligned}
 P_1^* - \frac{1}{2} &= \frac{2(1 - d_2)}{3 - 4d_2^2} - \frac{1}{2} = \frac{(1 - 2d_2)^2}{3 - 4d_2^2} \geq 0 \text{ if } d_2 \leq \frac{1}{2} \\
 \pi^* - \frac{1}{4} &= \frac{1 - d_2}{3 - 4d_2^2} - \frac{1}{4} = \frac{(1 - 2d_2)^2}{4(3 - 4d_2^2)} \geq 0 \text{ if } d_2 \leq \frac{1}{2}
 \end{aligned}$$

### *Case I - Scenario II.*



Similar to the above scenario, we derive the constraints that parameters  $d_1, d_2$  need to satisfy by substituting optimal prices  $P_1^*, P_2^*$  and  $R^*$  as shown in (4.14) into the conditions in this sub-scenario:  $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ ,  $P_1 - d_1P_2 \leq 1 - d_1$ ,  $P_2 \geq 2R$ , and  $P_1 \in [0, 1]$ ,  $P_2 \in [0, 1]$ .

$$P_2 \geq 0 \Rightarrow d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2 \geq 0 \quad (4.32)$$

$$P_2 \geq 2R \Rightarrow d_2 \leq \frac{d_1}{2} \quad (4.33)$$

$$\begin{aligned} & (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1) \\ \Rightarrow & \frac{(-1 + d_1)d_1(4d_2^3 + d_1^2(-1 + 2d_2) + d_1(d_2 - 6d_2^2))}{2(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} > 0 \\ \Rightarrow & 4d_2^3 + d_1^2(-1 + 2d_2) + d_1(d_2 - 6d_2^2) < 0 \end{aligned} \quad (4.34)$$

since constraint (4.32) above and  $d_1 \leq 1$

$$\begin{aligned} & 0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) \\ \Rightarrow & \frac{(-1 + d_1)(4d_1d_2^2 - 4d_2^3 + d_1^3(-1 + 2d_2) + d_1^2(d_2 - 2d_2^2))}{2(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} \leq 0 \\ \Rightarrow & 4d_1d_2^2 - 4d_2^3 + d_1^3(-1 + 2d_2) + d_1^2(d_2 - 2d_2^2) \geq 0 \end{aligned} \quad (4.35)$$

since constraint (4.32) above and  $d_1 \leq 1$

On the other hand, constraint (4.33)  $d_2 \leq d_1/2 \Rightarrow d_1^2 + 2d_1d_2 - 4d_2^2 \geq 0$ , so combining this with constraint (4.32) gives us the following:

$$\begin{aligned} P_1 - 1 &= -\frac{(1 + d_1)(d_1^2 + 2d_1d_2 - 4d_2^2)}{2(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} \leq 0 \\ \frac{P_1 - d_1P_2}{1 - d_1} - 1 &= -\frac{d_1^2 + 2d_1d_2 - 4d_2^2}{2(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} \leq 0 \end{aligned}$$

Therefore, (4.32), (4.33), (4.34), and (4.35) are the constraints that parameters  $d_1, d_2$  need to satisfy in this scenario in order that the optimal solutions (4.14) and optimal expected revenue (4.15) are feasible.

Under the constraints (4.32), (4.33), (4.34), and (4.35), one can show that both the optimal full information price and the maximum expected revenue in this scenario are also more than 1/2 and 1/4 respectively. In fact, we have

$$\begin{aligned}\pi^* - \frac{1}{3+d_1} &= \frac{(1+d_1)^2(d_1-2d_2)^2}{4(3+d_1)(d_1^3-d_1^2(-2+d_2)+d_1d_2-4d_2^2)} \\ &\geq 0 \quad \text{given condition (4.32)}\end{aligned}\tag{4.36}$$

On the other hand,

$$\frac{1}{3+d_1} \geq \frac{1}{4} \text{ for } 0 \leq d_1 < 1.$$

Therefore,  $\pi^* \geq 1/4$ . Since  $P_1^* = \pi^*/2$ , so  $P_i^* \geq 1/2$ .

*Case I - Scenario III.* Similarly, we obtain the constraints that  $d_1, d_2$  need to satisfy by plugging the optimal solutions  $P_1^*, P_2^*$ , and  $R^*$  into the conditions  $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ ,  $P_1 - d_1P_2 \leq 1 - d_1$  and  $P_2 < 2R$ . Note that  $P_1^* = 2P_2^*/(1 + d_1)$ ,  $R^* > P_2^*/2$ ,  $P_1 \in [0, 1]$ , and  $P_2 \in [0, 1]$ .

In fact,

$$\begin{aligned}&(d_1 - d_2)P_1^* - d_1P_2^*(1 - d_2) + 2d_2R(1 - d_1) \\ &> P_2^*[\frac{2}{1+d_1}(d_1 - d_2) - d_1(1 - d_2) + d_2(1 - d_1)] \\ &= P_2^*[\frac{2}{1+d_1}(d_1 - d_2) - (d_1 - d_2)] \geq P_2^*[d_1 - d_2 - (d_1 - d_2)] \geq 0;\end{aligned}\tag{4.37}$$

$$\frac{P_1^* - d_1P_2^*}{1 - d_1} - 1 = \frac{-1}{3 + d_1} < 0;$$

$$(d_1 - d_2)P_1^* - d_1P_2^*(1 - d_2) = \frac{(1 - d_1)(d_1 - d_1d_2 - 2d_2)}{3 + d_1} \leq 0 \Rightarrow d_1 \leq \frac{2d_2}{1 - d_2}\tag{4.38}$$

Thus,  $d_1 \leq 2d_2/(1 - d_2)$  is the parameter constraint in this scenario.

It is easy to see that the optimal full information price  $\frac{2}{3+d_1}$  and the maximum expected revenue  $\frac{1}{3+d_1}$  in this scenario are greater than or equal to those values in the situation where there is only the REG channel i.e.  $1/2$  and  $1/4$  respectively, since we have the following:

$$P_1^* = \frac{2}{3+d_1} > \frac{2}{3+1} = \frac{1}{2}, \text{ and } \pi^* = \frac{1}{3+d_1} > \frac{1}{3+1} = \frac{1}{4}$$

## CHAPTER 5

### CUSTOMER PREFERENCES AND OPAQUE INTERMEDIARIES

#### 5.1. Introduction

The selling of services (hotel rooms, rental cars, airline seats, etc...) online has dramatically changed how service firms reach customers, with online travel sales now exceeding offline (or traditional sales channels). Adding online selling channels provides firms an opportunity to expand the market and achieve a finer consumer segmentation (Zettelmeyer 2000, Geysken et al. 2002). We use an online choice experiment to assess the impacts of price and service quality upon customer purchase behavior across multiple online distribution channels, using hotel booking as an illustration. The online distribution channels include a regular full information channel like Expedia, Travelocity and Orbitz; an opaque posted price channel like Hotwire or Lastminute; and an opaque bidding channel like Priceline's name-your-own-price (NYOP). Hotwire and Priceline, unlike the regular full information channel, offer customers opaque products with aspects of the service provider concealed until the transaction has been completed. For instance a customer booking a hotel room through Hotwire does not know the identity or exact location of their non-refundable choice property until after their purchase. Similar to the opaque posted price model of Hotwire, Priceline offers opaque services but without posted prices. On Priceline's NYOP, consumers post bids for the opaque service and have to wait for the service provider to accept or reject their offer. Opaque selling has become popular in service selling as it allows firms to sell their full priced products to regular brand loyal customers while simultaneously selling opaquely to non loyal customers

at discounted prices. We use a Multinomial Logit (MNL) model to assess the impacts of a hotel's attributes (such as price, star rating, guest rating and so on) along with consumer's own characteristics (loyalty to certain hotel brand, income level, business and leisure travel frequencies etc.) upon purchase behavior.

The rest of the paper is organized as follows: In section 5.2 we briefly summarize the related literature and motivate the study. Section 5.3 outlines the questionnaire and focuses on the experimental design. Section 5.4 describes the experimental data collected and summarizes model results. We conclude in section 5.5.

## **5.2. Literature Review**

Our online experiment is motivated by the adoption of opaque intermediaries for the sale of services. Opaque selling mechanisms have recently received increasing attention in the academic literature. Anderson (2009) provides a detailed description of Priceline's NYOP model and illustrates methods for determining optimal bid policies on Priceline. Anderson and Xie (2012) use a nested logit model in combination with logistic regression and dynamic programming to illustrate how a service firm can optimally set opaque prices for a hotel using Hotwire.com. Anderson and Xie (2010) develop a stylized model of a monopolist simultaneously selling a product via the three selling channels, a full information channel, a posted price opaque channel and an opaque channel with bidding. Anderson and Xie (2010) show how opaque channels naturally segment customers as a function of their product valuations. This experiment is a

natural extension of this prior research and empirically studies how consumers would trade off the desire of obtaining full or more information about a product with the higher prices associated with that information as well as the incremental revenue generated for the supplier by adding opaque listings to a regular full information listing.

### 5.2.1 Opaque Selling

Using opaque selling in addition to regular selling naturally segments customers as it allows firms to sell their differentiated products at higher prices to regular brand loyal customers while simultaneously selling opaque products to non loyal customers at discounted prices. Opaque selling enables firms to facilitate price discrimination and expand the market. These advantages has also popularized online opaque channels and stimulated the growing interest on opaque selling in the academic literature.

Most of the early research has been focused on models similar to Price-line's NYOP bidding mechanism (Hann and Terwiesch 2003; Fay 2004; Spann et al. 2004). Recently, research has looked at using opaque channels in a multi-channel selling environment. Fay (2008) uses a traditional Hotelling model to study a game between two service providers selling products to two types of customers (*loyals* and *searchers*) on both an opaque posted price channel and a traditional distribution channel. Fay shows that opaque selling benefits the monopoly service provider when customers have heterogenous values for products. Similar to Fay (2008), Jiang (2007) also develops a Hotelling type model to illustrate how a firm should price on regular full information channels versus

opaque channels. Jiang compares opaque selling and regular selling (selling full-information products), providing insight on when to implement opaque selling. Jerath et al. (2010) compare opaque selling with last-minute direct selling and obtain the conditions under which opaque selling is preferred. Wang et al.(2009) develop a two period game theoretic model of a supplier using both regular posted price full information channels as well as a NYOP channel to reach heterogeneous customers. Fay (2009) examines a game between two firms, and each one can sell either through the NYOP mechanism or the posted price channel like Hotwire. In his model, customers have different frictional costs on bidding on a NYOP retailer but have the same reservation value for products. Cai et al. (2009) investigates the potential benefits to a NYOP retailer by adding a retailer-own list-price channel. They compare two bidding scenarios, single-bid and double-bid, in both single-channel and dual-channel situations.

Priceline's NYOP mechanism is commonly viewed as a reverse auction (Ding et al. 2005; Fay and Laran 2009; Fay 2009). While there is an extensive body of research on the use of auctions very little has focussed on the simultaneous use of auctions and posted price selling, with exceptions including Etizon, Pinker and Seidmann (2006); Van Ryzin and Vulcano (2004 ); Huang and Sosic (2011) and Caldentey and Vulcano (2007) and Huh and Janakiraman (2008). Firms can use auctions to reach customers whom may not otherwise purchase, as posted prices may be too high. Conversely auctions potentially dilute revenue as customers willing to pay posted (full prices) may purchase (at lower prices) via the auction. The opaque nature of Priceline's NYOP model helps to avoid this dilution.

Anderson and Xie (2010) is the only paper that investigates a firm using two opaque (posted and bidding) channels simultaneously with regular full information posted price channels. Instead of assuming two or more exogenous customer segments (i.e. business and leisure) with the opaque channels targeted at the leisure or price sensitive segment) Anderson and Xie develop endogenous consumer segments where consumers strategically choose channel(s) by maximizing their surplus.

There are some other closely related research which consider various forms of product differentiation and price discrimination. Fay and Xie (2008) define a new form of product termed as *probabilistic goods* under which customers have a probability of getting any one of several distinct goods. Customers do not know the identity of the good until after the purchase, which is analogous to opaque selling. By introducing probabilistic goods, firms are able to reduce the mismatch between capacity and demand under the circumstance of uncertain customer preferences structure. Another similar concept *flexible products* was introduced by Gallego and Phillips (2004) and Gallego et al. (2004). By selling a *flexible product*, firms have the flexibility of assigning one out of a set of alternative products to the customer if there are excess inventories for that product. Like probabilistic goods, flexibility products are also offered at a discount, which provides a new dimension for firms to segment a market as well as improve capacity utilization.

An opaque product is also somewhat comparable to damaged goods in Deneckere and McAfee (1996) as products available on opaque channels lack some critical information before the transaction is finished. Deneckere and McAfee (1996) show that firms could have a Pareto improvement in profits through sell-



ing intentionally damaged goods along with high-quality goods to heterogeneous consumers. They model a monopolist firm segmenting the market in two cases by offering damaged goods.

In this study, we aim to investigate the impact of product factors (including the hotel's attributes like price, star rating, previous guest rating, winning chance of bidding as well as the consumer's own characteristics like household gross income, travel frequency and loyalty to a specific hotel chain) upon the purchase decision and as such deploy experimental discrete choice models. To our knowledge, our paper is the first empirical study to understand opaque selling along with regular selling through experimental choice analysis. We also attempt to empirically show that despite potential cannibalization of regular full information sales by opaque offerings, total expected revenue increases as the supplier attracts a larger market of price sensitive customers.

### **5.2.2 Experimental Choice Analysis**

In the past four decades, conjoint analysis has become the most widely applied method for measuring and analyzing consumer behavior and preferences. Green and Srinivasan presented a review of conjoint analysis in 1978 (Green and Srinivasan 1978) and updated it with new developments and related methods in 1990 (Green and Srinivasan 1990). Integrating and applying theory and methods from probabilistic discrete choice theory and the design of discrete multivariate statistical experiments; experimental choice analysis is a relatively recent type of conjoint analysis and has received considerable of attention from researchers in various areas. Louviere (1992) offers a review

on the state of art in experimental choice analysis. Verma et al. (1999) refers this as the choice experiments-based *probabilistic discrete choice analysis* (DCA). They provide guidelines for designing and conducting DCA studies for services as well as the steps for DCA applications based on choice experiments, which are illustrated by using an actual application in the pizza delivery Industry. Iqbal et al. (2003) implements experimental choice analysis to understand consumer choices and preferences in Transaction-based e-services. Pullman and Thompson (2003) develop an model by combining an experimental choice analysis-based optimal product design model from marketing with a simulation model investigating capacity and demand management strategies from operations management. They apply this model to determine the profit-maximizing capacity management strategy for a service network and test it using the data from an actual ski resort service.

In a typical choice experiment, respondents are asked to choose their preferences among a set of alternatives with different combinations of levels of multiple attributes in a hypothetical but realistic choice scenario. Louviere and Woodworth (1983) is one of the first papers that proposed the concept of experimental choice analysis and illustrate a variety of empirical examples that implemented the MNL model as the modeling approach. McFadden (1986) provides a comprehensive overview on uses of economic choice theory and choice models, especially MNL, in the analysis of experimental market research data. As in many experimental choice analysis related research (Iqbal et al. 2003; Louviere and Woodworth 1983; Elrod et al. 1992; Verma et al. 1999; Verma et al. 2001; Swait and Andrews 2003; Shankar et al. 2008), we implement the MNL model to analyze the experimental data and measure the impacts of alternatives' attributes on consumer preferences. In our study, we design the choice experiment and

estimate the resulting MNL model to understand consumer preferences among different types of online channel listings as well as investigate the potential revenue gain by adding opaque listings to the regular full information listing.

### **5.3. Online Choice Experiment**

The online choice experiment is designed to evaluate characteristics of a hotel and the sales channel that impact purchase behavior. Respondents are asked to select amongst product offerings in addition to a no-purchase option. Each respondent, in addition to the no purchase option sees three listings: a regular full information listing similar to what one would see at an online travel agent or suppliers website, a posted price opaque travel site and an opaque bidding travel site (subsequently referred to as REG, OPQ and BID respectively). Prior to seeing the choice scenarios respondents are asked to read educational information about the three forms of online selling.

#### **5.3.1 Attributes**

Respondents are shown ten choice scenarios with each scenario having four alternatives. The first three alternatives include hotels from three different online distribution channels (REG, OPQ, and BID) with the fourth the no-purchase option. We set the hotel attributes as realistic as possible in order to imitate the actual hotel booking experience. A sample choice scenario is shown in Figure 5.1. In an effort to communicate the characteristics of the channels we label the choices *Full Information*, *Partial Information* and *Partial Information with Bidding*

(REG, OPQ, and BID respectively).

The REG listing has three attributes: chain scale or star rating, price and traveler review score. The star rating of the hotel has three levels: 3,  $3\frac{1}{2}$  and 4 star. Prices have four levels: 0.86, 0.93, 1.15 and 1.3 times the average price for a city-star pair. Table 5.1 summaries average prices for the city-star pairs. Guest ratings or review scores have four levels : 3.5, 4, 4.5 and 4.8 points on a 5 point scale. The OPQ listings also have the same three attributes as REG. OPQ listings also have  $4\frac{1}{2}$  star hotels in addition to 3,  $3\frac{1}{2}$  and 4 star. OPQ prices have four levels (85%, 75%, 65% and 55%) with each levels a % of the corresponding city-star average price(Table 5.1). OPQ listings also have a *no review score* review score level in addition to the 3.5, 4, 4.5 and 4.8 levels used on REG listings. Similarly BID alternative is characterized by three attributes. The star rating of BID listings is the same as OPQ (3,  $3\frac{1}{2}$ , 4 and  $4\frac{1}{2}$ ). Similar to OPQ, BID prices are discounted average city-star pairs with levels 55%, 45% and 35% of the city-star averages. The third BID attribute is the probability that your bid is accepted, this attribute has two levels 50% (i.e. a good chance) and 95% or great chance. Consistent with Priceline's opaque bidding channel no review scores are shown for BID listings. The attributes (and levels) are summarized in Table 5.2.

#### 5.4. Data Description and Analysis

In this section, we provide some descriptives of the experimental data which help us understand some basic consumer purchase behavior among the full information channel and opaque channels before the full investigation using the choice modeling approach in the next section. We collected 5310 choice-set re-

Table 5.1: Average prices per room/night

	3 star	3½ star	4 star	4½ star
Los Angeles	\$135	\$161	\$245	\$321
Chicago	\$142	\$209	\$280	\$335
Atlanta	\$118	\$146	\$205	\$253
New York City	\$192	\$284	\$349	\$451
Washington DC	\$254	\$297	\$359	\$470
Seattle	\$139	\$151	\$186	\$193
Austin	\$131	\$160	\$218	\$288
Denver	\$121	\$159	\$215	\$199
Baltimore	\$148	\$174	\$250	\$340
Charlotte	\$105	\$125	\$168	\$190

Table 5.2: Attributes &amp; their levels

Alternative	Attribute	Levels
Full Information	Star rating	3, 3½, 4
	Price multiplier	0.86, 0.93, 1.15, 1.3
	Guest rating	3.5, 4, 4.5, 4.8
Partial Information	Star rating	3, 3½, 4, 4½
	Prices	55%, 65%, 75%, 85%
	Guest rating	shown, not shown
	Guest rating if shown	3.5, 4, 4.5, 4.8
Partial Information with Biding	Star rating	3, 3½, 4, 4½
	Prices	35%, 45%, 55%
	Chance of winning	95%, 50%

sponses from a sample of 531 respondents. Table 5.3 summarizes the demographic characteristics of the sample.

As shown in Table 5.4, 30.72% choices are the BID listings as opposed to

Table 5.3: Socio-demographic characteristics of the sample

	Population %
<i>Gender</i>	
Female	52.17
Male	47.83
<i>Age (years)</i>	
< 17	0.36
18 – 24	11.41
25 – 29	9.24
30 – 34	8.70
35 – 44	15.76
45 – 54	26.81
55 – 64	24.64
65+	3.08
<i>Household size (persons)</i>	
1	21.01
2	37.32
3	18.12
4	14.67
5	5.98
6+	2.90
<i>Marital status</i>	
Married or living with a partner	57.79
Single	41.67
Rather not say	0.54
<i>Gross household income (dollars)</i>	
< 25,000	24.28
25,000 – 34,999	15.76
35,000 – 49,999	17.75
50,000 – 74,999	19.93
75,000 – 99,999	10.70
100,000 – 124,999	5.80
125,000 – 149,999	2.53
150,000+	3.26
<i>Education</i>	
Some high school	3.08
High school / GED	20.47
Vocational / Technical	4.17
Some college	25.72
2-year college degree	11.59
4-year college degree	22.28
Master's degree	10.69
Doctoral degree	0.91
Professional degree (J.D., M.D.)	0.91
Other	0.18

15.16% REG listings, 25.57% OPQ hotel listings and 28.55% indicating they would not purchase from the options shown.

Table 5.4: Market share (%) by alternative

Alternative	REG	OPQ	BID	NONE
Percentage of choices %	15.16	25.57	30.72	28.55

Table ?? displays the market share (or choice percentages) in percentages by price and star combinations for each alternative. As you can see from the total market share in terms of only star rating or only price that 3 stars and lower prices are more popular options. This implies price plays a very important role in consumers choice behavior as lower stars are usually associated with lower prices. However, if you fix either the star rating or the price level and vary the other, the market share is not necessarily strictly decreasing in that attribute, which means consumers need to consider some other factors while making their choices. Thus, motivating our use of choice modeling to understand and quantify attribute impacts.

### 5.4.1 Customer Choice Modeling

Discrete choice models have been employed to study consumer choice behavior and understand consumer's brand preferences, market structure as well as product attributes. Coretjens and Gautschi (1983) provides a general survey of discrete choice models in marketing and a systematic introduction to discrete choice modeling theory. During the past 40 years, the multinomial logit (MNL) model has been the most popular discrete choice model used by marketing re-

Table 5.5: Market share (%) by star and price combination

Star rating	Price related attributes				
	Price multiplier				
REG hotel listing	0.86	0.93	1.15	1.3	Total by star rating
3	2.37	2.18	1.13	1.22	7.00
3.5	2.02	1.47	0.92	0.60	5.01
4	1.02	0.89	0.70	0.64	3.24
Total by price multiplier	5.40	4.54	2.75	2.47	
	Price				
OPQ hotel listing	55%	65%	75%	85%	Total by star rating
3	2.98	2.37	2.05	1.98	9.38
3.5	2.47	2.09	1.60	1.19	7.34
4	2.18	1.30	2.05	0.94	5.33
4.5	1.02	0.87	0.00	0.49	3.52
Total by price discount	8.64	6.63	5.71	4.60	
	Price				
BID hotel listing	35%	45%	55%		Total by star rating
3	4.60	2.81	1.86		9.27
3.5	3.39	2.90	2.52		8.81
4	3.48	2.34	1.66		7.48
4.5	2.32	1.60	1.24		5.16
Total by price discount	13.79	9.64	7.29		



searchers (e.g. Guadagni and Little 1983, Krishnamurthi and Raj 1991, Gönül and Srinivasan 1993, Fader and Hardie 1996).

In a discrete choice model, an individual decision maker (consumer) makes a choice among a feasible set of alternatives. The decision is often made by maximizing random utilities of the alternatives. The MNL model is the most commonly used random utility model. For a greater detail in discrete choice modeling or the MNL model, please refer to McFadden (1981) or Ben-Akiva and Lerman (1985).

In an MNL model, we usually consider a universal set of alternatives denoted by  $C$ . The set of alternatives for consumer  $n$  is denoted by  $C_n \subset C$ .  $U_{ni}$  denotes the random utility of alternative  $i$  for consumer  $n$ , which can be divided into two additive components:  $U_{ni} = V_{ni} + \varepsilon_{ni}$ , where,  $V_{ni}$  is the deterministic (representative) component.  $\varepsilon_{ni}$  is the random component and  $\varepsilon_{ni}$  s are independent and identically Gumbel distributed.

Each alternative  $i$  in a choice set  $C_n$  is often characterized by a finite set of observable attributes  $x_{ni1}, x_{ni2}, \dots, x_{nik}$  in the following way:

$$V_{ni} = \beta' \mathbf{x}_{ni} \quad (5.1)$$

where  $\mathbf{x}_{ni} = (x_{ni1}, x_{ni2}, \dots, x_{nik})'$  and  $\beta_i = (\beta_{i1}, \beta_{i2}, \dots, \beta_{ik})'$  is the vector of unknown parameters that need to be estimated.

Every consumer is a utility maximizer, so the choice probability of alterna-

tive  $i \in C_n$  is given by

$$\begin{aligned}
P_n(i) &= Pr[U_{ni} \geq U_{nj}, \text{ all } j \in C_n \text{ for } \forall n] \\
&= \frac{e^{V_{ni}}}{\sum_{j \in C_n} e^{V_{nj}}} \\
&= \frac{e^{\beta'_i \mathbf{x}_{ni}}}{\sum_{j \in C_n} e^{\beta'_j \mathbf{x}_{nj}}} \tag{5.2}
\end{aligned}$$

We estimate the parameters  $\beta_i$  and obtain the choice probabilities by applying maximum likelihood method.

In our MNL, the respondent makes a choice among the four alternatives in the choice set  $C = \{\text{REG}, \text{OPQ}, \text{BID}, \text{No Purchase}\}$ . The utility associated with the first three alternatives for respondent  $n$  is given by

$$U_{ni} = \beta'_i \mathbf{x}_{ni} + \varepsilon_{ni} \tag{5.3}$$

Where,  $i = 1, 2, 3$  represents the alternatives REG, OPQ and BID respectively;  $\mathbf{x}_{ni}$  are the vectors including attributes for alternative  $i$  and the characteristics of consumer  $n$ ;  $\beta_i$  are the corresponding vectors of parameters to be estimated using the MNL model.

By (5.2), the probability  $P_n(i)$  respondent  $n$  chooses alternative  $i$  is given as the follows:

$$P_n(i) = \frac{e^{\beta'_i \mathbf{x}_{ni}}}{\sum_{j=1}^{j=3} e^{\beta'_j \mathbf{x}_{nj}} + e^{U_{00}}} \tag{5.4}$$

where,  $i = 1, 2, 3$  represents the alternatives REG, OPQ and BID respectively.  $U_{00}$  represents the utility of "No Purchase" choice. The "No Purchase" choice

probability is

$$P_{00} = \frac{e^{U_{00}}}{\sum_{j=1}^{j=3} e^{\beta'_j x_{nj}} + e^{U_{00}}}. \quad (5.5)$$

Note that using an MNL model requires alternatives in the choice set to have the independence of Irrelevant Alternatives (IIA) property which holds that changes in the choice set do not affect the ratio of choice probabilities of any two alternatives. As one can see that in equation (5.2),  $\sum_{i \in C_n} P_n(i) = 1$ , the IIA property implies if some alternatives are removed or added from the choice set  $C_n$ , the choice probabilities  $P_n(i)$  will be proportionally revised so that they can still be summed up to be 1 (Ratliff et al. 2008). Specifically, in our situation, when we add either the OPQ hotel listing or the BID hotel listing (or both) to the REG hotel listing, we can compute the proportionally shifted new choice probabilities of each alternative and so the potential incremental expected revenue. We discuss this in detail later on as we investigate the incremental revenue to firm from utilizing opaque listings in addition to regular full information listings.

As one can see in (5.4) and (5.5), we are facing an issue of properly estimating the utility of the "No Purchase" choice ( $U_{00}$ ). We assume that there are enough hotels in the market such that "No Purchase" consumers actually purchase from other hotels (competitors). Under this assumption, analogous to Ratliff et al. (2008) which provides a recapture heuristic for estimating unconstrained demand from airline bookings, we use the average market share from aggregate data from Smith Travel Research ([www.str.com](http://www.str.com)) to proximate the "No Purchase" utility. Specifically we assume that we know or can estimate a hotel's market share (the total of the three listings on the three channels) denoted by  $MS_h$ , then the competitors' market share is  $MS_c = 1 - MS_h$  and this is the simpli-

fied estimate of the likelihood of the "No Purchase" choice  $P_{00}$ . Therefore substituting into equations (5.4) and (5.5) provides an estimate for "No Purchase" utility  $U_{00}$  which can be solved from the equation below

$$\begin{aligned} e^{U_{00}} &= \frac{MS_c}{MS_h} \sum_{j=1}^{j=3} e^{\beta'_j x_{nj}} \\ &= \frac{1 - MS_h}{MS_h} \sum_{j=1}^{j=3} e^{\beta'_j x_{nj}}. \end{aligned} \quad (5.6)$$

A similar estimation method was also implemented Meterelliyo et al. (2009) and Ferguson et al. (2009).

Substituting (5.6) and  $MS_c = 1 - MS_h$  into (5.4), we have the choice probabilities for all three alternatives  $i = 1, 2, 3$  (REG, OPQ, BID respectively):

$$P_n(i) = MS_h \frac{e^{\beta'_i x_{ni}}}{\sum_{j=1}^{j=3} e^{\beta'_j x_{nj}}} \quad (5.7)$$

## 5.4.2 Model Estimation Results and Analysis

In this section, we provide our MNL model estimation results in Table 5.6 and discuss how hotel attributes and consumer characteristics influence the choice behavior.

In Table 5.6, *Business frequent*, *Leisure frequent*, *Loyal to hotel* and *Income* are the four characteristics of respondents which have statistically significant impacts on their choice preference. Respondents were asked how often (weekly, monthly, quarterly, semi-annually, annually, less than once a year and never) they stay in hotels while traveling for business and leisure purposes respectively. We define frequent travelers (coded as 1) as those traveling either weekly

Table 5.6: MNL parameter estimation results

Variable Name	Parameter	Standard	P-value
	Estimate	Error	
Business frequent (REG)	1.8180	0.3101	0.0000
Business frequent (OPQ)	1.7182	0.2976	0.0000
Business frequent (BID)	0.6609	0.3181	0.0377
Leisure frequent (REG)	0.4656	0.2367	0.0491
Leisure frequent (OPQ)	1.0532	0.2085	0.0000
Leisure frequent (BID)	0.7824	0.2122	0.0002
Loyal to hotel (REG)	0.6251	0.1757	0.0004
Income (REG)	0.1692	0.0243	0.0000
Income (OPQ)	0.0594	0.0216	0.0059
Income (BID)	0.0504	0.0207	0.0149
REG constant	0.9635	0.4157	0.0205
OPQ constant	1.9570	0.1423	0.0000
BID constant	1.4886	0.2366	0.0000
REG price	-2.2751	0.2387	0.0000
OPQ price	-2.9436	0.2955	0.0000
BID price	-3.1986	0.3833	0.0000
BID chance of winning	0.5159	0.0624	0.0000
REG guest rating	0.0892	0.0798	0.2636
OPQ guest rating if shown	0.1782	0.0740	0.0160
OPQ guest rating (shown or no shown)	-0.5950	0.3232	0.0656
3.5 star	-0.2646	0.0504	0.0000
4 star	-0.6620	0.0530	0.0000
4.5 star	-1.0840	0.0647	0.0000

or monthly, all others infrequent (0). The coefficient estimates for the *Business frequent* variable among the three alternatives implying frequent business travelers prefer REG hotel listing to the other two. On the contrary, the frequent leisure travelers favor the OPQ alternative. Indicating business travelers have a preference for known hotel locations (perhaps at the expense of price) whereas

leisure travelers may forgo the exact location/name in exchange for a discount.

Respondents were asked if they belonged to a hotel chain loyalty program (and if so which one). These loyalty program members were then shown listings from chains that they were loyal to in addition to hotels from other chains. The dummy variable *Loyal to hotel* is coded as 1 if the REG listing is from a hotel within the respondents loyalty program and 0 otherwise. Its positive coefficient estimate indicates consumers who are in the loyalty program are more likely choose a listing from that chain (on the REG channel).

As one would expect, lower prices increase purchase likelihood. We use channel specific price effects to capture varying price sensitivities across the three channels, with all three channels having negative coefficients and impacts of price increasing with loss of information ( $REG < OPQ < BID$ ). Similarly, a higher chance of winning gives the higher utility to the BID alternative (*Chance of winning* variable is coded to be 1 if it is 95% and 0 if it is 50%). We can also use the channel specific price coefficients to estimate channel specific price elasticities, using the coefficients to estimate the percent change in purchase probability (of a channel) as a function of the percent change in price of that channel. The elasticity estimates are -1.85 for REG, -1.38 for OPQ and -.90 for BID. These elasticities are all negative as anticipated. They are also decreasing with opacity, this is a function of the already low opaque channel prices and while price does impact opaque channels a price change on the full information channel is more likely to impact share of that channel (as it moves opaque or no-purchase guests to the full information channel).

Guest ratings also have channel specific effects. For OPQ ratings have two coefficients, an indicator for listed as well as coefficient for the continuous guest

rating score. So for OPQ the utility impact of this attribute is the combination of these two effects, e.g. if a guest rating of 3.5 is shown, then the value to the utility is  $3.5 \times 0.1782 + (-0.5950) = 0.0287$ . This value is bigger than zero i.e. the value of the guest rating not shown. This tells us showing guest rating attracts consumers more than not showing it. Similarly, we can compute the value added to the utility for guest rating being 4, which is 0.1179, and so the marginal increment percentage to the utility when guest rating increases from 3.5 to 4 given other attributes remaining the same is  $(0.1179 - 0.0287)/0.0287 = 309.72\%$ . Table 5.7 provides the marginal utility increment percentages for both REG and OPQ alternatives when guest rating variable increasing one level. As we can see that the marginal utility increment percentages for the OPQ alternative are bigger than those for the REG alternative, which implies increasing guest rating has a bigger impact on consumers favoring the OPQ alternative than favoring the REG alternative. We believe this is because without knowing the exact identity of the hotel listing, previous guests' evaluations of the hotel become more valuable to consumers. Another finding is the utility increment percentage from increasing guest ratings decreases while guest rating gets bigger, and so this shows once the guest rating is high enough (e.g.  $\geq 4$ ), increasing guest rating will not have as big of the impact on consumers choice as before.

Table 5.7: The marginal utility increment percentages with increasing guest rating

Increasing guest rating	Marginal utility increment percentage %	
	OPQ	REG
3.5 to 4	309.72	14.29
4 to 4.5	75.59	12.50
4.5 to 4.8	25.83	6.67

From the estimates of the star coefficients we conclude that 3 stars are the most popular, which is consistent with the market share statistic shown in Table 5.5.

Table 5.8: Nested logit model fit diagnostics

$LL$	-6705.311
$LL^*$	-7195.937
Chi-squared[20]	981.253
Prob [chi squared>critical value]	0.000

Table 5.8 summarizes model fit characteristics. The  $LL$  value is the log-likelihood of the estimated model and  $LL^*$  is the log-likelihood for a reference model which has alternative specific constants only, i.e. the market shares predicted by the model are what are in the data. To determine whether the estimated model is superior to its reference model, the  $-2(LL - LL^*)$  value obtained is compared to a Chi-square statistic with degrees of freedom (20 in our model) equal to the difference in the number of parameters estimated for the two models (Hensher et al. 2005). As seen in Table 5.8 that the  $p - value$  (i.e. Prob [chi squared>critical value]) is 0.000, which is less than the level of  $\alpha = 0.05$ , and so we reject the null hypothesis that the estimated model is no better than the reference model.

### 5.4.3 Incremental revenue from opaque booking channels

This section illustrates the determination of incremental revenue for a service provider from adding opaque listings to their traditional full information listings. Assume a hotel has the flexibility to list rooms on any of the three chan-



nels: regular full information channel, opaque partial information channel and opaque partial information with bidding channel. The REG and OPQ listings are priced at  $p_1$  and  $p_2$  respectively, and the BID hotel listing is given a price  $p_3$  along with a certain winning probability to mock the bidding process.

The hotel lists rooms on all three channels and receives a market share  $MS_h$ . The hotel only receives a fraction (denoted by  $d$ ) of the demand for the opaque listings presented by the choice probability from the MNL model as given their opaque nature some of these transactions are at competing (same star, same similar location) hotels. Therefore, given the attributes of the three hotel listings, characteristics of the customer  $n$ , and fraction of the demand for the opaque listings  $d$ , the total expected revenue the hotel can obtain from these three listings is given by:

$$\pi_1 = p_1 P_{n1} + \sum_{j=2}^{j=3} d p_j P_{nj} \quad (5.8)$$

Where,  $P_{nj}$ ,  $j = 1, 2, 3$  are given in (5.7). If the hotel decides to only list a room on the regular full information channel, then by the IIA property mentioned previously, the choice probability  $P_{n1}$  for the REG alternative needs to be proportionally adjusted so that the sum of it and the choice probability of the *No Purchase* alternative is still one. Thus, we have

$$P_{n1} = \frac{e^{\beta'_1 x_{n1}}}{e^{\beta'_1 x_{n1}} + e^{U_{00}}}, \quad (5.9)$$

where  $U_{00}$  is given in (5.6). So the corresponding total expected revenue that the hotel can now obtain is

$$\pi_2 = p_1 P_{n1}, \quad (5.10)$$

where  $P_{n1}$  is given by (5.9).

If the hotel adds an OPQ listing to the REG listing, then similarly, the IIA prop-

erty implies the new choice probabilities  $P_{n1}$  and  $P_{n2}$  given by the following

$$P_{ni} = \frac{e^{\beta'_i \mathbf{x}_{ni}}}{\sum_{j=1}^{j=2} e^{\beta'_j \mathbf{x}_{nj}} + e^{U_{00}}}, i = 1, 2 \quad (5.11)$$

where  $U_{00}$  is given in (5.6). Therefore, the total expected revenue that the hotel can now have is

$$\pi_3 = p_1 P_{n1} + d p_2 P_{n2}, \quad (5.12)$$

where  $P_{n1}$  and  $P_{n2}$  are given by (5.11).

Similar to the above, but instead of the OPQ listing, the hotel adds the BID listing to the REG listing, then the total expected revenue that the hotel can get is

$$\pi_4 = p_1 P_{n1} + d p_3 P_{n3}, \quad (5.13)$$

where

$$P_{ni} = \frac{e^{\beta'_i \mathbf{x}_{ni}}}{e^{\beta'_1 \mathbf{x}_{n1}} + e^{\beta'_3 \mathbf{x}_{n3}} + e^{U_{00}}}, i = 1, 3$$

and  $U_{00}$  is given in (5.6).

Hence, adding an OPQ listing to a REG listing gives a potential revenue increment percentage of  $(\pi_3 - \pi_2)/\pi_2$ ; adding a BID listing to a REG listing gives a potential revenue increment percentage of  $(\pi_4 - \pi_2)/\pi_2$ ; and adding both opaque listings to a REG listing provides a potential revenue increment percentage of  $(\pi_1 - \pi_2)/\pi_2$ . Next, we use the following examples to illustrate how to compute these potential incremental revenues.

**Example 1** Consider a 3 star Hilton hotel priced at \$177 (the price multiplier is 1.15), located in Time Square, New York City, with guest rating 4.5, displayed on the regular full information channel. The respondent  $n$ , a member of the loyalty program of Hilton (i.e.HHONORS), is a frequent traveler for both business

and leisure purposes and with a household gross income of less than \$100,000. The service provider lists the hotel room on the opaque posted price channel at a price of \$100 per room/night (65% of average). The service provider can also list the hotel room on the opaque bidding channel with a 50 % chance that a bid 55% of average is accepted. We estimate the hotel's market share (required to estimate no-purchase utility) using data reported by Smith Travel Research (STR). STR reports average 2010 market share (by channel) as follows: online travel agents 4.6%, supplier website (e.g. Hilton.com) 17.5% and opaque online travel agent at 2% with other channels comprising the remaining 75.9%. We can approximate the hotel's demand on regular full information channel to be the sum of the online travel agent and the supplier's own website (23.1%) with opaque travel sites at 2%. Hence, the hotel's market share is approximately  $MS_h = 4.6\% + 17.5\% + 2\% = 24.1\%$ . With seven 3 star hotels in Time Square area, we approximate the fraction ( $d$ ) of the demand that the opaque listings of this hotel gets is  $1/7$ . Therefore, knowing the attributes of these three hotel listings, the consumer's own characteristics and the fraction parameter, along with the MNL model estimation results, we use the equations (5.8), (5.10), (5.12) and (5.13) to calculate the potential revenue increment percentages as shown in Table 5.9. In **Example 2** we raise the opaque price to \$115 per room/night, and in **Example 3**, the prices on the REG, OPQ and BID channels are changed to \$143 per room/night (the price multiplier is 0.93), \$115 per room/night (75%), and \$54 per room/night (35%) with a 95% chance of being accepted.

Table 5.9 is the revenue increment for a specific example, and Table 5.10 displays the average incremental revenues from adding an OPQ listing to a REG listing (averaged across all possible attribute levels). Similarly, Table 5.11 shows these same average incremental revenues from adding an opaque bidding chan-

Table 5.9: Revenue Increment from adding opaque listings (%) for specific examples

	Example 1	Example 2	Example 3
Adding an OPQ listing to a REG listing	6.29	4.91	4.91
Adding a BID listing to a REG listing	1.15	1.17	3.39
Adding both OPQ and BID listings to a REG listing	6.53	4.64	2.26

nel. Table 5.12 shows the highest and lowest average revenue increment percentages from adding both opaque listings to a REG listing for each REG price.

Examples 1-3 as well as Tables 5.10-5.12 show the potential revenue gains provided by the natural customer segmentation provided by adding opaque listings to full information listings. However, appropriate prices need to be set on these three channels in order to balance the revenue gain from having extra opaque demand and the revenue loss from diluting the revenue that we could have received from selling a full priced product through the regular channel. As seen in Example 1 (Table 5.9) the balance between the revenue gain and loss is well achieved as adding two opaque listings has the largest incremental revenue, whereas in Example 2 adding two opaque listings is worse than just one opaque listing. This revenue loss in Example 2 is attributed to the higher opaque posted prices (than Example 1) now with some OPQ customers becoming opaque bidders (at lower bids versus the opaque posted price).

If full information prices and opaque prices are not sufficiently separated then opaque selling fails to separate customers (into appropriate segments) and is unable to generate substantive revenue gains. With low full information prices very little gain is possible by adding opaque, with the potential for revenue dilution. Maximal customer segmentation is achieved as full information prices increase and opaque prices decrease. However, one might notice that

incremental revenues decrease with increasing OPQ prices but increase with increasing opaque bidding prices. This is a result of BID prices being lower than the OPQ prices. In fact, by adding the OPQ listing, the prices are high enough such that the negative impact to the revenue by lowering prices can be offset by the increased choice probabilities. Thus, the lower the price the bigger the price difference to the REG price which creates a better chance to attract more customers without diluting the REG channel demand. On the contrary, adding the BID listing with low prices, the extra demand can not compensate for revenue losses (from decreased REG market share).

As shown in Table 5.12, the service provider obtains the largest incremental revenue in the presence of all three channels, as if prices are set appropriately a finer degree of customer segmentation is possible (versus single or dual channels). Thus, implementing more opaque channels in addition to the regular channel and with an appropriate price structure will generate greater revenue than just using one opaque channel to a regular full information channel (not even mention just operating the full information channel alone).

Table 5.10: Average incremental revenue from adding an OPQ listing to a REG listing (%)

OPQ price	REG price multiplier			
	0.86	0.93	1.15	1.3
0.55	7.34	8.48	14.28	20.63
0.65	6.81	7.75	12.70	18.22
0.75	6.10	6.86	10.97	15.65
0.85	5.38	5.98	9.32	13.21

Table 5.11: Average incremental revenue from adding a BID listing to a REG listing (%)

BID price	REG price multiplier			
	0.86	0.93	1.15	1.3
0.35	-0.87	-0.67	0.75	2.55
0.45	0.24	0.45	1.85	3.58
0.55	0.88	1.08	2.37	3.94

Table 5.12: Incremental revenue from adding both OPQ and BID listings to a REG listing (%)

	REG price multiplier			
	0.86	0.93	1.15	1.3
Highest value	8.07	9.49	16.56	24.39
Lowest value	3.63	4.32	8.52	12.99


## 5.5. Summary and Managerial Insights

Selling through opaque posted channels (e.g. Hotwire.com) or /and opaque bidding channels (e.g. Priceline.com) along with regular full information channels (e.g. Expedia.com, Hilton.com) provides the service provider an opportunity to segment consumers, expand the market, and increase the total revenue. On the one hand, consumers who are less sensitive to prices and loyal to a brand (or simply prefer to know the hotel name/location) will choose to book the room through the regular full information channel. On the other hand, price sensitive consumers who are brand agnostic will exchange information for price and book through opaque channels. Therefore, adding opaque channels to an existing regular full information channel enables the service provider capture more

consumers without diluting the revenue (assuming prices are set efficiently). Through our stated preference choice experimental procedure we have been able to empirically validate these potential revenue gains.

We use an MNL choice model to study the consumers purchase preferences among these three channels and find that the regular full information channel is more attractive to those less price sensitive frequent business travelers. In contrast, the leisure frequent travelers favor the hotel listed on the opaque channels than the listings on the regular channel. As we expect that consumers with lower income tend to be more price sensitive and value listings on the opaque channels. Consumers who are members of loyalty programs tend to exhibit brand preferences resulting in an increase in the utility of the full information channel. As expected favorable guest review scores will increase the likelihood of being chosen with a higher marginal impact on opaque listings versus full information listings.

Given the options below, which hotel would you choose?

ATTRIBUTES	ALTERNATIVES		
	Full Information	Partial Information	Partial Information with Bidding
Hotel Name	Courtyard Atlanta Midtown/Georgia Tech	Any Qualifying Hotel	Any Qualifying Hotel
Location Information	1132 Techwood Drive, Downtown, Atlanta, GA	Downtown, Atlanta, GA	Downtown, Atlanta, GA
Star Rating	3.5 Star	3.5 Star	4.5 Star
Amenities	Detailed Amenities 		
Price per Room/Night	\$136	\$110	\$89
Guest Rating	4.5	4.8	50% chance of winning
Which would you choose?	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Next

0%  100%

Figure 5.1: Sample choice scenario



## CHAPTER 6

### SUMMARY

As a new revenue management technique, opaque selling has become popular in service selling as it allows firms to sell their differentiated product at higher prices to regular brand loyal customers while simultaneously selling to non-brand loyal customers at discounted prices.

The first two chapters in the dissertation provide an overview on opaque selling mechanism and its related academic literature. In Chapter 3, a nested logit model is estimated on a data from firms selling hotel rooms at Hotwire.com to understand the customers' choice preferences on hotels located in Washington D.C. This research is the first choice based revenue management model that uses market level demand data (versus a single firm's sales data) to estimate the underlying demand models. Demand is modeled as a function of a property's prices using the result from the nested logit model and a choice-based dynamic programming model is developed with pricing as the decision variable. Two stochastic dynamic programming formulations is proposed, one with daily fixed prices and one with fully dynamic prices. We provide a complete characterization of optimal dynamic prices and a partial characterization for optimal daily fixed prices. We show numerically that optimal fixed prices are consistent with the properties of dynamic prices. We then use these two characterizations to estimate revenue impacts from daily fixed versus fully dynamic pricing policies. Although it might be easier for firms to implement daily fixed pricing policy, dynamic pricing offers better revenue increment opportunities. This formulation can be extended to the situation where the firm simultaneously release inventory across numerous channels - both opaque like Hotwire

(e.g. Priceline's Name-Your-Own-Price) and fully transparent (e.g. Expedia or Marriott.com). Thus, an interesting direction for future research is to consider consumer choice behavior both across service providers as well as channel selection.

In Chapter 4, we have developed a stylized model of when and how to deploy an opaque selling strategy in concert with regular full information pricing. Unlike previous research which usually assumes an exogenous consumer separation into regular consumers and opaque consumers we endogenously model this channel selection process as a function of prices and channel characteristics (opacity). We have shown that even in the face of capacity constraints firms should be simultaneously using opaque channels in concert with regular channels whereas historically focus has been on using opaque channels to sell distressed or otherwise unsellable inventory (surplus capacity). The simultaneous use of opaque selling with regular full information selling effectively segments consumers - allowing firms to sell at higher prices to higher valuation/brand loyal consumers and at lower prices to lower valuation/brand agnostic shoppers via opaque channels and increase firm revenues. Instead of considering a single service provider, a potential model extension could be incorporating competition among service providers in a game setting while each has the opportunity to sell through all three types of distribution channels.

As a natural extension of the research in Chapter 4, Chapter 5 empirically studies how consumers would trade off the desire of obtaining full or more information about a product with the higher prices associated with that information as well as the incremental revenue generated for the supplier by adding opaque listings to a regular full information listing. Through our stated prefer-

ence choice experimental procedure we have been able to empirically validate the potential revenue gains by adding opaque channels to an existing regular full information channel (assuming prices are set efficiently). We use an MNL choice model to study the consumers purchase preferences among these three channels and find that the regular full information channel is more attractive to those less price sensitive frequent business travelers. In contrast, the leisure frequent travelers favor the hotel listed on the opaque channels than the listings on the regular channel. As we expect that consumers with lower income tend to be more price sensitive and value listings on the opaque channels. Consumers who are members of loyalty programs tend to exhibit brand preferences resulting in an increase in the utility of the full information channel. As expected favorable guest review scores will increase the likelihood of being chosen with a higher marginal impact on opaque listings versus full information listings. This research is limited by the difficulty of simulating the bidding procedure of the opaque bidding channels. In order to do a choice experiment, we list the bid price in stead of consumers submitting their own bids and only provide two winning probabilities. So in our future research, one direction is to improve our experiment design and offer more winning probabilities, so that we can imitate the bidding scenarios better. Another direction that might be worth to explore is to design the experiment so that allows the respondents to bid as in real life, but still in a way that enables us to do the choice analysis. We hope that this could be a good addition to both literature of the experimental choice analysis and empirical study of opaque selling.

## CHAPTER 7

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